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RELATIVE DISTRIBUTION AND ABUNDANCE OF ELEMENTS IN THE LOWER CHROMOSPHERE

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ABSTRACT

I. By combining the heights to which various lines in a given multiplet are observed in the flash spectrum with the multiplet intensity formulae, it has been possible to derive for several elements the laws of density distribution in the lower chromosphere. It is found that the density gradients at 2000 km for atoms of very different types such as Ti II, Fe I, and H are nearly equal. This seems to indicate that turbulence and not selective radiation pressure is responsible for keeping the elements so well mixed. In general, the density gradients are considerably less than the theoretical density gradient of H in an isothermal atmosphere of $5000°$ K.

II. The relative density distribution of other elements is discussed. Attention is called to the peculiar distribution of atoms emitting certain lines of a given element, as compared with the distribution of atoms of the same elements emitting other lines. It seems probable that Mg II and Ca II are supported more by selective radiation pressure than by turbulence, as their density gradients are considerably smaller than the average.

III. The relative abundance of various elements was determined by Russell's method with the exception that the calibration of the scale between various spectral regions was effected by the use of Kronig's theoretical formulae for the transition probabilities of different multiplets in a triad; furthermore, Pannekoek's modification of the ionization formula to allow for chromospheric deviations from thermodynamical equilibrium has been used. Within the limits of error, the chromospheric abundances, with the exception of H and He, parallel those of the reversing layer found by Russell, there being no marked differences depending on atomic weight. This is further evidence that turbulence must be responsible for the support of the chromosphere.

IV. Certain aspects of Menzel's recent study of the chromosphere are discussed. It is found that his density gradients are in substantial agreement with those derived in this paper. The observational basis for his conclusions concerning temperature gradients is questioned, as are also certain of his conclusions with respect to relative abundances.

Recently two papers have been published from this observatory dealing with the spectrum of the chromosphere. The first, in *Astro-*

physical Journal, **71,** 1, 1930, gave the details of 3250 lines of the flash spectrum photographed by means of a concave grating without slit. Neutral iron was represented by 1222 lines, or 37.6 per cent of the total, and hence a study was made in *Astrophysical Journal*, **72,** 146, 1930, of correlations between heights, intensities, excitation potentials, etc. The paper herewith presented has been in preparation for some time. After the first three sections were practically completed, Menzel's "Study of the Solar Chromosphere"[1] reached our hands. It seemed worth while to supplement his admirable discussion by publishing an independent determination from entirely different material which covers a greater range of wave-lengths at both ends of the spectrum than was available to Menzel, and is more homogeneous, especially in the determination of intensities and heights.

I. DENSITY DISTRIBUTION OF *Fe* I, *Ti* II, *Cr* II, *Sc* II AND *Fe* II

In addition to the 1222 lines of *Fe* I, the concave grating spectra contain 226 lines of *Ti* II, 91 of *Fe* II, 88 of *Cr* II, and 48 of *Sc* II which represent the basic data of this section.

General method.—The procedure of the present discussion is: first, to calculate the relative amount of energy emitted along each one of a number of lines of sight, by atoms of a given type arranged in the chromosphere according to an arbitrarily postulated density distribution. Then these relative amounts are compared with the relative amounts of energy observed to be emitted along each of the lines of sight (deduced from our knowledge of multiplet structure, as will be explained below). By trial and error, a density distribution is found which agrees with the observations.

The reliability of the final results will depend on the accuracy with which the heights have been derived. The heights in the concave grating spectra given in Table V of the first paper were derived from measures of the angular lengths of the chromospheric arcs on a fixed plate. The method is explained in the second publication.[2] Quite simply, it was done by measuring the chords from tip to tip, or half-chords subtended by these arcs (L_1K_1, L_2K_2, etc., in Fig. 1). The heights derived from the chromospheric arcs were

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[1] *Lick Observatory Publications*, **17,** Part I, 1931.   [2] *Op. cit.*, p. 150.

calculated on the assumption that the exposure was instantaneous precisely at second or third contacts. As a matter of fact, the exposure with the 1905 concave grating spectrum was 1.4 seconds. Allowance has been made for this in the computations which are fully described on page 11.

Under the ideal conditions outlined above, the flash spectrum would be photographed when the limbs of the sun and moon were
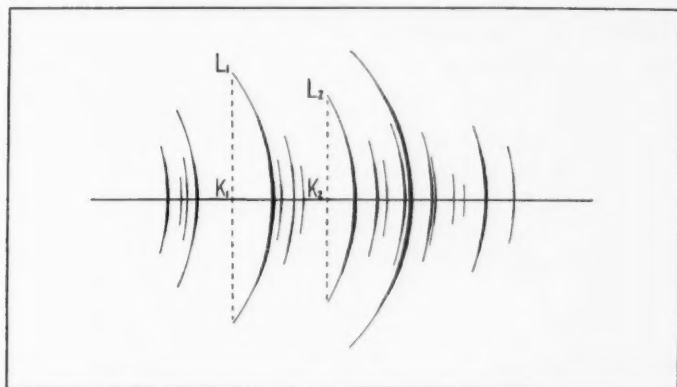
Fig. 1

tangent as in Figure 2. Chromospheric vapors extend above the photosphere, the densest layers lying closest to the photosphere; two of these layers are indicated in the diagram. Suppose the point P_1 corresponds to the tip of the chromospheric arc of the spectral line L_1 of measured height 1500 km, and the point P_2 corresponds to the tip of the line L_2 of height 1000 km. The measures of the half-chords P_1M_1 and P_2M_2 and a knowledge of the semi-diameters of the sun and moon give the heights BH and BF. Now, the emitting atoms contributing to the tips of the two lines under consideration lie along lines of sight which are perpendicular to the plane of the paper and which pass through P_1 and P_2. We shall designate these lines of sight as P_1P_1' and P_2P_2'. It is evident from the diagram that the line of sight P_2P_2' lies tangent to the chromospheric layer assumed to have a height of 1000 km and P_1P_1' lies tangent to the chromospheric layer of 1500 km. Now the tip of the spectral line represents a certain definite intensity, manifestly depending on the threshold

sensitivity of the photographic plate. When the number of atoms along the line of sight reaches a certain definite number, then the radiation is sufficiently intense to record itself on the photograph. Hence, in a limited region of wave-lengths near λ_0, the amount of
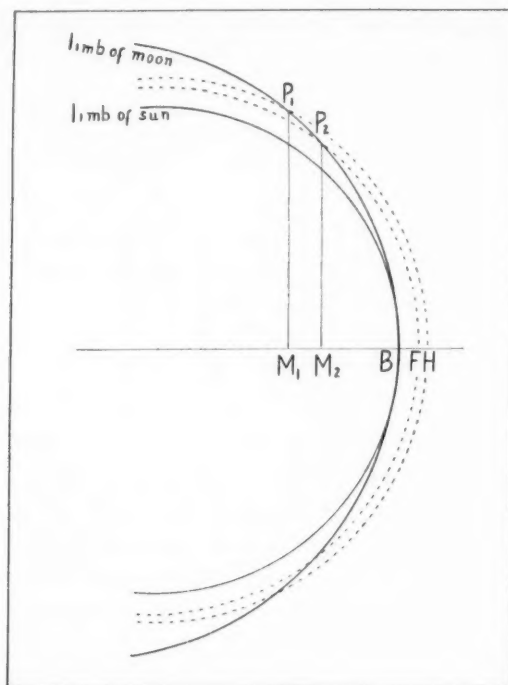
Fig. 2

emitted energy contributing to the tips of each of the lines is a constant, $E(\lambda_0)$. Then, if the spectral lines L_1 and L_2 lie close to λ_0, the amount of energy of wave-length L_1 emitted by atoms along line $P_1 P_1'$ is exactly equal to that of wave-length L_2 emitted by atoms along line $P_2 P_2'$, both being equal to $E(\lambda_0)$.

If the lines L_1 and L_2 belong to the same multiplet, and F_1 and F_2 are ratios predicted by the theoretical intensity formulae, where

$$F_1 = \frac{\text{Number of atoms contributing to the Line } L_1}{\text{Total number of atoms contributing to the multiplet}},$$

then F_1/F_2 gives the relative numbers of atoms emitting the lines L_1 and L_2 in any unit of volume. Hence, along the line of sight P_2P_2', there must be F_1/F_2 as many atoms emitting L_1 as are emitting L_2. Thus along the line of sight P_2P_2', corresponding to a height of 1000 km., there will be $(F_1/F_2)E(\lambda_0)$ of L_1 radiation, whereas along the line of sight P_1P_1', corresponding to 1500 km, there is only $E(\lambda_0)$ of L_1 radiation. (The fundamental assumption is made that the amount of self-absorption in the tip of the crescent L_1 is the same as that in the tip of the crescent arc L_2. This is an approximation which is probably nearly correct.) Therefore, in descending from a height of 1500 km in the chromosphere to one of 1000 km, there is an increase in the total number of the emitting atoms amounting to F_1/F_2. H. H. Plaskett has suggested a more rigorous wording:

In any given cm³ of the chromosphere, there are F_1/F_2 as many quanta being emitted per second in the line L_1 as are being emitted in the line L_2. Since the same number of quanta per second are required to produce the tip of L_1 and the tip of L_2, it follows that the number of atoms in the line of sight for the tip of L_2 is to the number of atoms in the line of sight for the tip of L_1 as F_1/F_2.

Obviously, this method is applicable only when the atomic origin of the lines is known and when two or more lines arise from a multiplet in which the relative intensities of the lines are predictable by the theoretical intensity formulae. Thus, the intersystem lines are of no use because no intensity formulae exist for such lines, and the lines of the Balmer series also cannot be used since no two lines belong to the same multiplet.

It is perhaps well to point out that the density gradients derived in this manner do not depend in any way on any particular calibration of the plate, so long as all the lines of a multiplet lie within a fairly narrow spectral region.

Example.—Consider, for instance, a multiplet consisting of five lines, the theoretical relative number of quanta emitted in each line being given below, together with the heights to which these lines are visible. Evidently there must be five times as many atoms emitting this multiplet along the 800-km line of sight as along that at 1500 km, because only one-tenth of all the quanta emitted in a unit volume are of wave-length c, whereas one-half of them are of wave-

length *a*. In other words, the density varies in such a manner that along the 800-km line of sight there are as many atoms emitting line *c* as are emitting line *a* along the 1500-km line of sight, which means that there are five times as many atoms along the 800-km line of sight as along the 1500-km line of sight. Similarly, there are ten times as many atoms along the 600-km line of sight as along the 1500-km line of sight.

Line	*a*	*b*	*c*	*d*	*e*	Sum
Observed height in km....	1500	1200	800	800	600	....
Theoretical relative number of quanta.............	100	50	20	20	10	200

Observations.—There are 21 *Fe* I multiplets, 17 multiplets of *Ti* II, 7 of *Cr* II, 6 of *Fe* II, and 5 of *Sc* II which may suitably be treated in a manner similar to that in the foregoing example. Thus, out of the total of 48 *Sc* II lines observed in the concave grating flash spectra, only 29 lines, belonging to 5 multiplets, are available for this purpose. The other 19 lines cannot be used because of a variety of reasons: 8 are singlets, 5 are intersystem lines, the rest belong to multiplets in which there are too many blended lines. In the case of *Fe* I, a number of lines cannot be used because their atomic origin is unknown. The results of this analysis are shown in Table I. E_h indicates the amount of energy emitted by atoms of the given type along lines of sight corresponding to height *h* given in kilometers. Thus, from the foregoing example we would have

$$\frac{E_{1200}}{E_{1500}} = 2 \quad \text{and} \quad \log \frac{E_{1200}}{E_{1500}} = 0.30 .$$

To obtain the values of $\log E_h/E_{h+\Delta h}$, we have used all the unblended lines of the multiplet and a graphical derivation.

It will be noticed that in the case of *Fe* I, there are available multiplets ranging in excitation potential from 0 to 4.6. In the case of the other elements the range is smaller, although there is a range of 2 volts in the excitation potentials of the *Ti* II data.

TABLE I

$$\text{LOG}\ \frac{E_h}{E_{h+\Delta h}}$$

Wave-Length	Excitation Potential	$h=300$ $\Delta h=50$	350 50	400 100	500 100	600 200	800 200	1000 200	1200 300	1500 500	2000 500
						Fe I					
3475	0.07						.21	.21	.32		
3600	0.99			.17	.17	.34	.34	.34	.29		
3720	0.07					.12	.12	.12	.18	.31	
3750	0.99					.26	.26				
3850	0.99					.47	.47	.40	.50		
3890	0.07						.17	.18	.30	.33	.27
4050	1.54							.31	.45		
4225	2.43			.12	.12	.25	.25				
4260	1.54			.42	.42	.66	.48	.44	.62		
4625	3.24		.85	.72							
4900	3.36	.30									
4920	2.84	.40	.36	.36	.24						
5430	0.99			.90	.40						
5620	3.24	.35	.35	.45	.35						
5700	4.24	.60	.26								
5850	4.58	.60	.60								
6150	2.57		.25	.40	.40						
6250	2.42			1.00	.50						
6250	2.19	.28	.28	.42	.40						
6300	3.63			.42	.50						
6650	2.36			.25	.25						
						Ti II					
3070	0.02							.16	.27	.38	
3230	0.02									.73	
3270	1.23								.62		
3330	0.02							.31	.48		
3360	1.22					.50					
3320	1.23							.38			
3380	0.02									.52	.52
3480	0.13						.50	.50			
3530	2.05						.50	.50			
3700	1.57					.26					
3750	1.57					.23	.23	.23	.36		
4310	1.16						.32	.25	.22	.32	
4300	1.08						.16	.16	.24	.40	
4430	1.08							.16	.24	.40	.40
4470	1.16							.20	.30	.50	.50
4560	1.22							.09	.14	.24	.24
4790	2.00				.10	.20					

TABLE I—Continued

Wave-Length	Excitation Potential	$h=300$ $\Delta h=50$	350 50	400 100	500 100	600 200	800 200	1000 200	1200 300	1500 500	2000 500
						Cr II					
3130	2.43							.20	.32		
3340	2.43						.70	.40	.35		
3400	2.43			.08	.08	.16		.29	.29	.43	
3740	3.09			.08	.08	.16					
4590	4.05			.13	.13	.22	.18	.13	.20		
4850	3.84	.38	.22	.23	.18						
5300	4.05	.40									
						Sc II					
3570	0.00						.27	.27	.27		
3640	0.00								.72		
4300	0.60					.13	.24	.24	.25		
4400	0.60				.20	.20	.33	.27			
5660	1.49					.40					
						Fe II					
4300	2.68					.15	.27	.20	.16	.12	.15
4540	2.82					.18	.32	.20			
4570	2.82					.09	.18	.18	.18	.26	
5300	3.18		.85	.80							
6250	3.88				.65	.55					
6430	2.88					.14	.16				

Table II presents a summary of Table I. Thus, the first item in column 3 gives the mean of six determinations of $\log E_{300} - \log E_{350}$ from the six different *Fe* I multiplets, together with the probable error derived from the residuals. That is to say, the energy emitted by *Fe* I atoms along lines of sight corresponding to a height of 300 km is 2.6 times the energy emitted along lines of sight corresponding to a height 350 km (since $\log^{-1} 0.42 = 2.6$). The parentheses around the last item in column 3 indicate that only two multiplets contributed to the mean. The data for *Fe* I have been divided according to the excitation potential of the lower term, because of the observation made by St. John[3] and confirmed by one of us[4] that the density gradient of *Fe* I depends on the excitation potential of the lower

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[3] *Astrophysical Journal*, **70**, 319, 1929.   [4] Mitchell, *ibid.*, **72**, 146, 1930.

TABLE II

h	Δh	Fe I				LOG $E_h/E_{h+\Delta h}$				Coefficient a for Fe I	Coefficient b for Ti II
		All E.P.	E.P.=0.0-0.1 Volts	E.P.=0.8-1.6 Volts	E.P.=3.0 Volts	Ti II	Cr II	Sc II	Fe II		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
300	50	.42 ± .04	......	......	.37 ± .03	......	(.39 ± .01)	......	......	+ .14 ± .03	......
350	50	.42 .06	......	......	.40 .06	......	......	......	......	.08 .07	......
400	100	.49 .06	......	(.50 ± .14)	.47 .07	......	.15 .04	.24 ± .06	(.72 ± .05)	.06 .14	......
500	100	.36 .04	......	(.33 .05)	.37 ± .04	......	.13 .02	......	.22 .05	+ .09 ± .07	......
600	200	.32 .05	.17 ± .02	.41 .07	......	.30 ± .05	(.19 .02)	.28 .02	.23 .03	......	......
800	200	.29 .04	.17 .02	.39 .05	......	.31 .04	.39 .12	.26 .01	.19 .01	......	+ .31 ± .19
1000	200	.29 .04	......	.38 .03	......	.24 .02	.26 .04	(.26 ± .01)	(.17 .02)	......	.17 .06
1200	300	.38 .04	.27 .04	.46 ± .05	......	.33 .02	.32 ± .03	......	(.21 ± .04)	......	.23 .06
1500	500	(.32 ± .01)	(.32 ± .01)	......	......	.45 .03	......	......	......	......	......
2000	500	......	......	......	......	.42 ± .05	......	......	......	......	+ .20 ± .07

term—atoms absorbing lines of high excitation potential being relatively more concentrated at small heights above the photosphere. A comparison of columns 4 and 5 confirms this observation.[5] In column 6 the value of $\log E_h/E_{h+\Delta h}$ corresponding to an excitation potential of three volts derived from all multiplets whose excitation potential is greater than 2 volts has been given as deduced by a least-squares solution of a set of linear equations of the form

$$\log \left. \frac{E_h}{E_{h+\Delta h}} \right]_{\substack{E.P.=3.0 \text{ volts} \\ \nu=18,600}} + a(E.P. - 3.0)$$

$$+ b(\nu - 18,600) \times 10^{-4} = \text{Observed value}, \quad (1)$$

where ν is the wave number $(10^8/\lambda)$ in the region of the multiplet. The coefficient of a was always positive, which would indicate, aside from the large probable errors, that even above 2 volts there is a dependence of density gradient on excitation potential. The coefficient b was sometimes negative and sometimes positive, but has relatively large errors. Hence, a second solution was made assuming $b=0$, and the derived values of the constants are given in columns 6 and 11. It is seen that an increase of 1 volt in excitation potential corresponds to an increase of about 25 per cent in the density gradient.

In the case of Ti II, the data were analyzed in a similar manner whenever more than four determinations of $\log E_h/E_{h+\Delta h}$ were available. In this case, on the contrary, the coefficient b seemed to be of considerable importance and a of no importance. A second solution was made, using the Ti II material and assuming $a=0$. The derived values of the constants are given in columns 7 and 12. The sign of b indicates that the density gradient decreases as the intensity of the radiation increases, the density gradient for multiplets at λ 3000 being two or three times as great as for multiplets at λ 4800; but it should be noted that the relative errors in the coefficient b are fairly large.

The distribution of Ti II seems to be very similar to that of Cr II and also to that of Sc II, as might be expected since these three ele-

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[5] Although Table II does not give the density distribution directly, the fact that the items in col. 5 are decidedly larger than those of col. 4 must indicate a steeper density gradient for the atoms of higher E.P.

ments have about the same atomic weight and about the same ionization potential with ultimate lines in the same region of the spectrum. On this account the three elements have been grouped together in the rest of this discussion.

Calculation of relative energy according to assumed density laws.— If we assume a density law, we can compute the resulting values of $\log E_h / E_{h+\Delta h}$. A comparison of the computed values with those observed shows whether the assumed law is a possible one.

In order to avoid confusion of ideas in the following, we use x as the height in the chromosphere above some arbitrary level, which is Milne's notation. The letter h has been reserved for the heights of individual spectral lines tabulated in the first paper, Table V.

A detailed explanation of the method of computation follows. The total number of emitting atoms T_R in a column of unit cross-section along any line of sight RR', say, in Figure 3 which passes x_R cm above the photosphere, is

$$T_R = 2 \int_0^\infty n_z d\zeta , \qquad (2)$$

where n_z is the number of emitting atoms per cubic centimeter. We assume a certain density law:

$$n_z = f(x) \qquad (3)$$

and express x in terms of ζ by means of the approximate relation[6]

$$x = \frac{\zeta^2}{13.9 \times 10^{10}} + x_R . \qquad (4)$$

So that, from equations (2), (3), and (4),

$$T_R = 2 \int_0^\infty f \left(\frac{\zeta^2}{13.9 \times 10^{10}} + x_R \right) d\zeta . \qquad (5)$$

Now atoms along all the lines of sight at a distance MP above the $\xi\zeta$-plane in Figure 3, namely, PP', QQ', RR', etc., contribute to the

[6] This is derived from the exact relation for points along the line RR':
$$(6.95 \times 10^{10} + x)^2 = \zeta^2 + (6.95 \times 10^{10} + x_R)^2 .$$

tip of the crescent arc of height 1000 km. Then to find a measure of the total energy emitted by atoms of a given type along all these lines of sight, it is necessary to find the relative contribution of atoms along each line of sight, PP', QQ', RR', etc. Thus the con-
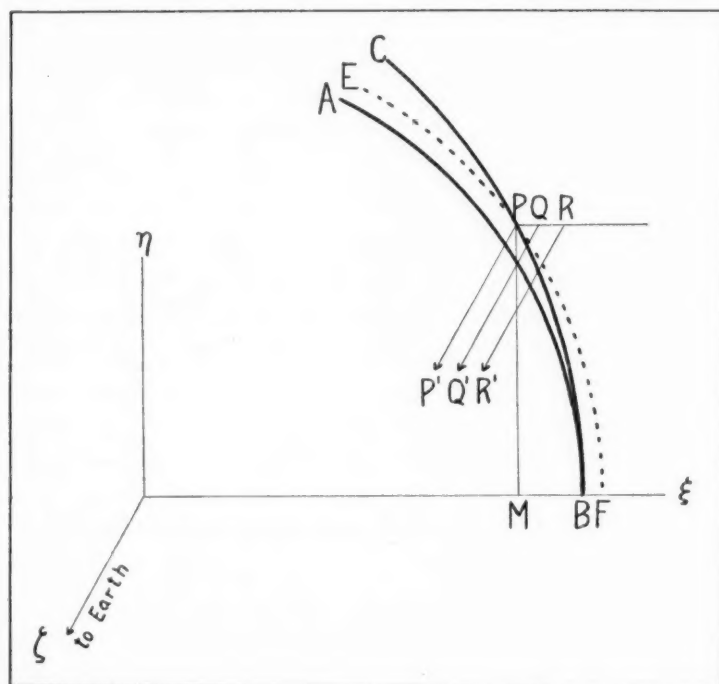
FIG. 3.—In the $\xi\eta$-plane: AB is the photosphere; CB is the projection of the limb of the moon; EF is a layer of the chromosphere 1000 km above the photosphere. In three dimensions: Equation of photosphere: $\xi^2+\eta^2+\zeta^2=(695,300\times10^5)^2$. Equation of projection of moon's limb: $(\xi+\Delta)^2+\eta^2=(728,200\times10^5)^2$, where $\Delta=(32,900-300t)\times10^5$ and $t=$ number of seconds before third contact. Equation of layer 1000 km above photosphere: $\xi^2+\eta^2+\zeta^2=(696,300\times10^5)^2$.

tribution of atoms along QQ' will be proportional to the product of T_Q and $(t_Q)^{0.8}$, where t_Q is the exposure time of QQ', the Schwarzschild exponent having been taken as 0.8. Thus by numerical integration we find, for instance,

$$E_{1000\times10^5}=\text{const.}\int_{OM}^{\infty}T_{1000\times10^4+s}(t)^{0.8}d\xi\,,\qquad(6)$$

where

$$s = (\xi - OM) \cos \tan^{-1} \frac{MP}{OM}$$

and

$$t = \frac{(\xi - OM)}{300 \times 10^5} \tag{7}$$

t being not greater than 1.4, since the moon passed over 300 km of the sun's surface per second and the maximum length of exposure was 1.4 seconds.

The value of $E_{1000 \times 10^5}$ given by equation (6) represents the total energy emitted, and takes no account of self-absorption. Since we are not dealing directly with values of E, but with the ratio of one to another, we simply assume that the self-absorption in the tips of the lines in a given spectral region is a constant.

In columns 2–7 of Table III, the observational material given in Table II has been rearranged. In columns 8 and 9 we have inserted for purposes of comparison, values for hydrogen and helium which we have derived from Figures 19 and 20 of a paper by Pannekoek and Minnaert.[7] With these values are to be compared values in columns 10–14, computed from assuming various density laws as indicated in the table where the unit of heights h is centimeters.

Discussion.—It should be noted that if we assume a law of density corresponding to complete support of atoms by radiation pressure, so that the number of atoms is proportional to $(x+x_0)^{-2}$, then the number of emitting atoms, n_2, is proportional to[8]

$$A(x+x_0)^{-2} + B(x+x_0)^{-3} . \tag{8}$$

[7] *Verh. d. Kon. Akad. v. Wet. t. Amsterdam*, **13**, No. 5, 1928.

[8] We are indebted to Professor Milne for these expressions. Equation (8) can be obtained by combining equations (13), (15), (21), and (23) of pp. 177–180 of his monograph in the *Handbuch der Astrophsyik*, Band **3**, 1930; equation (9) can be obtained by combining equations (13), (15), (21'), and (23'). The expressions have been derived on the assumption that the chromosphere is composed chiefly of the atom under consideration, and that the atom possesses only two energy states. Neither of these assumptions holds for our case, of course. In equation (8) x is the height in centimeters above the photosphere; x_0 is a constant of integration. In equation (9), μ is the fraction of the weight of the atom supported by the gas-pressure gradient; m is the mass of the atom; g is the acceleration due to gravity at the sun's surface; k is Boltzmann's constant; and T is the temperature of the chromosphere.

TABLE III

| $h \times 10^{-8}$ | $\text{LOG}\dfrac{E_h}{E_h+100\times10^5}$ Observed | | | | | | | | $\text{LOG}\dfrac{E_x}{E_x+100\times10^5}$ Computed on Various Assumptions (See Below) | | | | |
| | Fe I | | | | Ti II, Etc. | Fe II | H | He I | | | | | |
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
300..	.84	.....	.....	.77	(61)	.....	.....	(03)	.13	.043 K	.16	.23	.20
400..	.49	.....	(50)	.47	.16	(72)	.....	.....	.12	.....	.14	.22	.18
500..	.36	.....	(33)	.37	.17	.22	.07	.....	.10	.043	.14	.21	.17
600..	.16	.....	.21	.....	.14	.12	.....	.....	.09	.....	.14	.20	.16
700..	.16	.....	.20	.....	.13	.11	.....	.....	.07	.....	.13	.20	.15
800..	.15	.09	.20	.....	.16	.10	.....	.....	.06	.....	.13	.19	.14
900..	.14	.08	.19	.....	.15	.09	.....	.....	.06	.....	.13	.18	.13
1000..	.15	.09	.19	.....	.13	(09)	.07	(10)	.06	.043	.12	.18	.13
1100..	.14	.08	.19	.....	.12	(08)	.....	.....	.05	.....	.12	.17	.12
1200..	.13	.09	.16	.....	.12	(07)	.....	.....	.05	.....	.11	.16	.11
1300..	.13	.09	.15	.....	.11	(07)	.....	.....	.04	.....	.11	.15	.11
1400..	.12	.09	.15	.....	.11	(07)	.....	.....	.04	.....	.11	.15	.10
1500..	(07)	(07)	.....	.....	.10	.....	.07	.....	.04	.043	.11	.....	.10
1600..	(07)	(07)	.....	.....	.09	.....	.....	.....	.03	.....	.10	.....	.09
1700..	(06)	(06)	.....	.....	.09	.....	.....	.....	.04	.....	.10	.....	.10
1800..	(06)	(06)	.....	.....	.09	.....	.....	.....	.03	.....	.10	.....	.09
1900..	(06)	(06)	.....	.....	.08	.....	.....	.....	.03	.....	.10	.....	.09
2000..	.....	.....	.....	.....	.09	.....	.07	.....	.03	.043	.10	.....	.09
2100..	.....	.....	.....	.....	.08	.....	.....	.....	.03	.....	.10	.....	.09
2200..	.....	.....	.....	.....	.09	.....	.....	.....	.03	.....	.09	.....	.09
2300..	.....	.....	.....	.....	.08	.....	.....	.....	.03	.....	.09	.....	.08
2400..	.....	.....	.....	.....	.08	.....	.....	.....	.03	.....	.09	.....	.09

COL. 2. All excitation potentials of Fe I combined.

3. Fe I with excitation potentials of 0.0–0.1 volts.

4. Fe I with excitation potentials of 0.8–1.6 volts.

5. Fe I with excitation potentials of 3.0 volts.

6. Ti II, Cr II, and Sc II.

7. Fe II.

8. Values for $H\gamma$ of hydrogen derived from Pannekoek and Minnaert's paper.

9. Values for λ 4471 of helium, of low weight, from the same paper.

10. Assuming the number of emitting atoms is proportional to x^{-3}.

11. Assuming the number of emitting atoms is proportional to $e^{-Kx\times10^{-8}}$.

12. Assuming the number of emitting atoms is proportional to $e^{-2x\times10^{-8}} + 20e^{-4x\times10^{-8}}$.

13. Assuming the number of emitting atoms is proportional to $e^{-3x\times10^{-8}} + 50e^{-6x\times10^{-8}}$.

14. Assuming the number of emitting atoms is proportional to $e^{-2x\times10^{-8}} + 60e^{-5x\times10^{-8}}$.

Similarly, if we assume a law of density distribution corresponding to partial support by radiation pressure such that the number of atoms is proportional to $e^{-\frac{\mu mgx}{kT}}$, then for neutral atoms n_2 is proportional to something like[8]

$$e^{-\frac{\mu mgx}{kT}} + \tfrac{3}{2}Ce^{-\frac{2\mu mgx}{kT}} . \tag{9}$$

It was immediately evident upon calculation that no law of the form (8) will fit the observed data. Even with the most favorable assumptions with respect to the disposable constants, the maximum values obtainable for log $E_h/E_{h+\Delta h}$ are at most only about half of the minimum observed values (cf. col. 10 of Table III). Thus, as was to be expected, the possibility of the total support of Ti II, Fe II, etc., by radiation pressure is ruled out.

On the other hand, it is possible to fit all the data roughly with some form of equation (9). However, formula (9) is so flexible that a great variety of data may be fitted with it. The Ti II data, for example, are approximately fitted by the expression

$$n_2 \propto e^{-2x \times 10^{-8}} + 20e^{-4x \times 10^{-8}} \tag{10}$$

(see col. 12 of Table III). The Fe I data of E.P. = 0.0 are fitted within the errors of observation by the expression

$$n_2 \propto e^{-2x \times 10^{-8}} \tag{11}$$

(see col. 11 of Table III). The Fe I data of E.P. = 0.8–1.6 are fitted by the expression

$$n_2 \propto e^{-3x \times 10^{-8}} + 50e^{-6x \times 10^{-8}} \tag{12}$$

(see col. 13 of Table III).

In general, it may be said that aside from any theoretical formulae, all the data may be well represented by a density law of the type

$$n_2 \propto e^{-C_1 x} + C_2 e^{-C_3 x} . \tag{13}$$

Thus the Ti II data are better fitted by the expression

$$n_2 \propto e^{-2x \times 10^{-8}} + 60e^{-5x \times 10^{-8}} \tag{14}$$

than by expression (10) (see col. 14 of Table III).

[8] See note on p. 13.

If we assume that equation (9) is applicable to the Fe I data in Table III, then, comparing equations (9) and (11), we have for Fe I

$$\frac{\mu mgx}{kT} = 2x \times 10^{-8} ,$$

whence the fraction of the total weight supported by radiation pressure is found to have the improbably high value

$$1 - \mu = .99 .$$

But aside from such comparisons, columns 2–9 of Table III seem to contain evidence that selective radiation pressure as suggested by Milne is not the factor which is chiefly responsible for the support

TABLE IV

| | RELATIVE DECREASE IN DENSITY IN 100 KM | | | | | |
| HEIGHT IN KM | Observed | | | | Predicted in Isothermal Atmosphere | |
	Fe I Ultimate Lines	Fe I E.P. = 1 Volt	Ti II	H	Fe I	H
500.............	No data	.58	.48	.16	33.50	.60
1000.............	.20	.51	.42	.16	33.50	.60
2000...	.20	No data	.25	.16	33.50	.60

of the chromosphere. This evidence lies in the fact that *the values of the density gradients for all elements are of the same order.* Thus, the ultimate lines of Fe II lie in a region of the spectrum where the intensity of the solar radiation is only one-third of that in the region which contains the ultimate Ti II lines, yet the observed density gradients are the same within the limits of error. Again, the density gradient for hydrogen above 1000 km is only slightly smaller than that for other elements. This seems to indicate that *turbulence must be responsible for keeping the elements so well mixed.*

In Table IV, it is of interest to compare the actual decrease in densities at various heights for the different elements, as computed from their derived density laws, with the decreases expected in an isothermal atmosphere at 5000° K. It is seen that the observed density gradients are, in general, considerably less than the theo-

retical density gradient of hydrogen in an isothermal atmosphere of temperature 5000°.

Errors.—In the third edition of *Eclipses of the Sun*, pages 247–298, 1932, there are discussed from the observer's point of view the difficulties experienced in securing photographs of the flash spectrum. The problem is a complicated one because the moon has mountains and valleys projected against the edge of the sun, which in turn has prominences, while in addition an observer has few opportunities in his life to gain experience in technique. At the beginning of totality, if the exposure for the flash spectrum is made too early, the bright lines of the chromosphere will be seen at the edges of the photospheric spectrum, while if the exposure is delayed, the advancing moon will cover up the low-lying chromospheric lines. Fortunately, with the fixed plate the spectra themselves give definite information regarding the levels actually photographed. If low levels are attained, the lines of the flash spectrum are superposed on a strong continuous spectrum. At second contact, 75 per cent of the lines of the flash spectrum are covered by the advancing moon inside of 2 seconds, while only 15 per cent of the total number of lines are visible after 3 seconds from the beginning of totality.

It is interesting to determine how much error is introduced by the assumption that the shutter was closed precisely at the instant of third contact. Although it is well-nigh impossible to define accurately what is meant by "third contact," experience shows that the best method of photographing the chromosphere with a fixed plate is by observing the actual appearance of the flash spectrum in a direct-vision spectroscope. Let us assume that the exposure ended not more than 0.3 seconds too early or too late, in which interval the moon moved across 100 km of the chromosphere or photosphere. If the shutter was closed 0.3 seconds too late, then x, the true height, would be equal to $h-100$, and the items in columns 1–7 of Table III should be moved up one space relative to the items in columns 10–14. It is seen that our conclusions are not much modified by this assumption.

The errors introduced by an error in the exposure time are readily evaluated. Had the true exposure been 4 instead of 1.4 seconds as assumed, the items in column 10 would have been decreased, the

first by 0.01, the last few items by about 0.001. In columns 12–14, the first few items would have been decreased by 0.01 and the rest would have been unchanged. More difficult to evaluate are the errors in the tabulated heights due to irregularities in the moon's limb, disturbances in the chromosphere especially near the tips of the arcs, etc.

II. DENSITY GRADIENTS OF OTHER ELEMENTS

For the other elements in the chromosphere, it has been possible to derive only qualitative results on the density gradients. In the first of the two publications on the flash spectrum, reference was made to the fact that the estimated chromospheric intensities closely approximate the Rowland scale of intensities. This is true in general only for the neutral lines. Enhanced lines and also neutral lines of low excitation potential are stronger in the chromosphere than in the Fraunhofer spectrum. In the second of the two publications[9] attention was called to estimates which were possible above levels of 600 and 1000 km, respectively. Thus, for purposes of comparison, there are four separate series of intensity estimates on similar scales which are designated as I_R = Rowland, I_0, I_{600}, and I_{1000} which are estimates above the zero-level and above the 600- and 1000-km levels. The estimates of the two higher levels were made by Mitchell at the same time as he was engaged in estimating the flash intensities. The four separate estimates are given in Table V; the region of wave-lengths is from λ 3318 to λ 5183. I_0 and I_{600} were made from the same film (of the second flash) at the 1905 eclipse. The estimates I_{1000} were made from another film taken immediately before the second flash. It should be noted that the intensities I_{1000} are not exactly comparable with I_0 and I_{600} on account of differences in exposure time, development, etc.

An examination of the estimates in Table V gives the following interesting information:

Helium.—That most of the helium atoms which emit light of accessible wave-length are more than 600 km above the photosphere is shown by the fact that $I_{600} = I_0$ for both *He* I and *He* II lines. The maximum density of emitting atoms probably occurs between 600

⁹ *Op. cit.*, p. 154, 1930.

TABLE V

Wave-Length	Element	R	0	600	1000	Ht.
3318.03	Ti II	6	7	0	....	1000
3321.71	Ti II	4	9	1	?	1200
3322.91	Ti II	8²	20	5	2	1500
3326.75	Ti II	5	8	2	....	1000
3329.42	Ti II-Co I	8²	18	5	2	1500
3332.08	Ti II	3	12	3	?	1200
3335.18	Ti II	6²	15	4	?	1500
3336.30	Cr II-Fe I	4²	6	1	....	1000
3339.79	Cr II-Co I	3	7	3	....	1000
3340.33	Ti II	5²	15	6	3	1500
3341.88	Ti II-Fe I	8²	25	15	8	2000
3342.61	Cr II	3	7	?	....	1000
3343.74	Ti II-Fe I	4	8	?	....	1000
3346.73	Ti II-Cr I	5²	10 }	3	....	1000
3347.89	Cr II-Fe I	6²	7 }			1000
3349.00	Ti II-Cr I	9¹	20	10	5	1500
3349.41	Ti II	9²	35	25	18	2500
3353.74	Sc II	4	8	1	....	1000
3358.50	Cr II	4	12	4	2	1200
3360.35	Cr II	2	10	4	2?	1000
3361.24	Ti II-Sc II	5²	25	15	12	2500
3368.07	Cr II	5	18	10	6	1500
3368.92	Sc II	3	6	1	....	800
3372.84	Ti II	10²	30	20	15	2500
3380.25	Ti II-Fe I	9¹	15	8	4	1500
3382.72	Cr II	4	12	8	6	1500
3383.84	Ti II-Fe I	9¹	25	20	15	2500
3387.88	Ti II-Zr II	5	18	12	8	1500
3391.96	Zr II	2	8	2	....	800
3393.80	Cr II	2	4	1	....	800
3394.56	Ti II-Fe I	6²	15	10	6	1500
3399.30	Zr II-Fe I	5²	4	?	....	800
3402.43	Ti II-Cr II	3	8	3	?	1200
3403.35	Cr II-Ni I	6¹	12	6	3	1500
3407.21	Ti II-Ni II	4²	5	?	....	800
3408.81	Cr II	3	15	10	6	1500
3409.79	Ti II	2	5	?	....	800
3410.22	Zr II-Fe I	3²	4	?	....	800
3414.78	Ni I	15	7	?	....	1000
3421.25	Cr II	4	12	9	5	1500
3422.71	Cr II-Fe I	7¹	15	12	7	1500
3430.57	Zr II	1	5	?	....	1000
3433.34	Cr II	3	18	12	8	1500
3438.22	Zr II	2	7 }	?	....	800
3438.97	Mn II-Fe I	4²	4 }		....	800
3440.62	Fe I	20	12	6	3	1500
3441.01	Fe I	15	10	5	3	1500
3441.97	Mn II	6	20	12	8	1500
3444.30	Ti II	4	12	5	?	1200
3456.44	Ti II	3	10	?	....	800
3460.32	Mn II	4	18	12	6	1500
3461.46	Ti II	5	12	6	2	1200
3465.79	Ti II-Fe I	12⁴	10d	3	....	1000
3474.11	Mn II	4¹	15	10	6	1500
3477.17	Ti II	5	12	6	2	1200
3482.95	Mn II	5	15	10	5	1500
3488.68	Mn II	4	12	9	5	1200
3490.60	Fe I	10	8 }	8	4	800
3491.03	Ti II	5	10 }			1200
3495.68	Mn II-Co I	7¹	6 }	5	2	800
3496.18	Zr II-Y II	3¹	10 }			1200
3504.88	Ti II-Fe I	5²	18	12	5	1500
3510.87	Ti II	5	15	10	4	1500
3515.01	Ni I	12	7	2	....	800
3517.33	V II	3	6	2	....	800
3520.23	Ti II	2	10	4	....	1000
3524.50	Ni I	20	12	4	....	800
3530.72	Ti II	3	8	3	....	1000
3535.41	Ti II	4	20	8	2	1200
3545.23	V II	4	10	4	....	1000
3549.05	Y II	2	7	3	....	1000
3551.92	Zr II	1	7	2	....	800
3556.71	Zr II-V II	11⁴	15d	5	....	1000
3558.52	Sc II-Fe I	8	10	4	....	1000
3565.37	Fe I-Ti II	15²	12	7	2	1200
3567.73	Sc II	6²	8	3	....	1000
3570.14	Fe I-Mn I	21²	15	10	4	1200
3572.52	Sc II-Zr II	10²	18	12	5	1200
3573.39 } 3573.97	Ti II-Fe I	12⁵	7d	4	....	800
3576.38	Sc II	7²	15 }	10	5	1200
3576.89	Zr II	1	5 }			800
3578.73	Cr I	10	12	6	3	1200
3580.96	Sc II	5	10 }	15	10	1000
3581.22	Fe I	30	20 }			1500
3585.30	Fe I-Co I	12²	15 }	8	4	1200
3585.63	Fe I-Cr II	8²	15 }			1200
3586.85	Fe I	11²	5 }	4	....	500
3587.20	Ti II-Co I	9¹	8 }			800
3589.72	V II-Sc II	10²	10 }	3	....	800
3590.48	Sc II	4²	8 }			800
3592.00	V II	2	7	1	....	800
3596.03	Ti II	4	10	5	2	1200
3600.77	Y II	3	12	4	1	1200
3601.89	Y II	1	8	1	....	800
3603.84	Cr II-Fe I	7³	7	1	....	800
3605.33	Cr I-Fe I	16³	8d	1	....	800
3608.86	Fe I	20	12	6	1	1200
3611.06	Y II	2	8	2	....	800
3613.82	Sc II	7²	25	15	10	1500
3618.77	Fe I	20	12	8	4	1200
3619.42	Ni I	8	6	2	....	800
3624.39	Ni I-Ca I	10³	5d	2	....	800
3624.84	Ti II-Fe I	5	12	6	3	1200
3628.71	V II	2	5	1	....	800
3630.01	Zr II	1	4 }	15	....	800
3630.73	Sc II-Ca I	7²	20 }		10	1500
3631.48	Fe I	15	12	8	4	1200
3633.13	V II	2	6	3	....	1000
3634.25	He I-Fe I	6²	4	3	3	1200
3641.35	Ti II	4	15	10	5	1200
3642.74	Sc II-Ti I	12³	18	12	5	1200
3645.34	Sc II-La II	6²	12	5	2	1200
3647.81	Fe I	12	12	8	3	1500
3651.77	Sc II	4	10	5	1	1200
3659.70	Ti II-Fe I	10²	12d	8	....	1000
3661.28	H31	......	3	0	....	1000
3662.23	Ti II	5	15	10	?	1200
3663.42	H29-Fe I	9¹	7	4	?	1200
3664.66	Y II-H28	2	15	7	?	1500
3666.09	H27	......	12	10	6	1500
3667.74	H26	......	12	10	7	1800
3669.46	H25	......	18	12	10	1800
3671.33	H24-Zr II	0	18	13	10	2000
3673.82	H23	......	18	14	10	2000
3676.34	H22-Fe I	6	20	15	12	2200
3677.79	Cr II-Fe I	13¹	15	4	....	1000
3679.34	H21	......	25	15	12	2500
3682.82	H20	......	25	15	12	2500
3685.25	Ti II	10	80	55	30	6000
3686.83	H19	......	30	18	13	3000
3691.62	H18	......	35	25	14	3000
3694.13	Fe I-Ti I	10¹	12	1	....	800
3697.21	H17	......	40	30	15	3500
3703.89	H16	......	45	35	18	4000
3704.97	He I	......	4	3	2	1200
3705.57	Fe I	9	8 }	8	2	1200
3706.11	Ca II-Ti II	9²	20 }			1200
3709.32	Fe I	8	8	1	....	800
3710.34	Y II	3	20	2	....	800

TABLE V—Continued

WAVE-LENGTH	ELEMENT	R	0	600	1000	HT.
3712.06....	H15	.....	50	45	20	5000
3719.94....	Fe I	40	35	15	12	2000
3722.00....	H14	.....	55	50	22	5600
3734.45....	H13	....}				
3734.91....	Fe I	40}	70	50	25	5600
3737.00....	{Ca II-Ni I / Fe I}	8² } / 30}	40	25	13	2000
3741.64....	Ti II	4	25	12	8	2000
3743.49....	Fe I-Cr I	12⁵	10d	2	....	1000
3745.78....	Fe I	14²	30	15	10	2000
3748.25....	{Ti II / Fe I}	1 / 10}	20	6	2	1500
3749.50....	Fe I	20	15	?	....	1000
3750.25....	H12	.....	70	55	30	6000
3757.66....	Ti II-Cr I	4	10	5	....	1000
3758.25....	Fe I	15	10	6	....	1000
3759.33....	Ti II	12	70	45	30	6000
3761.33....	Ti II	7}				
3761.88....	Ti II	3}	70	50	35	6000
3763.80....	Fe I	10	15d	6	....	1000
3765.49....	Fe I	6	6	2	....	800
3766.68....	Zr II-Fe I	4²	5	....	....	750
3767.15....	Fe I	8	10	5	....	1000
3770.72....	H11	.....	80	70	40	6000
3774.38....	V II	3	20	4	....	800
3776.02....	Ti II	2	7	2	....	800
3783.47....	Ni I-	8²	8	1	....	750
3788.66....	V II	2	18	2	....	800
3794.76....	La II	1	6}			800
3795.01....	Fe I	8	7}	3	....	800
3798.02....	H10	.....	90	80	45	6000
3805.32....	Fe I	6	4	1	....	750
3813.10....	Fe I	7²	7	3	....	1000
3814.53....	Ti II-Fe I	7²	10d	1	....	700
3815.82....	Fe I	15	15	6	2	1500
3819.63....	He I	.....	10	10	8	5000
3820.43....	Fe I	25	20	8	3	1500
3821.79....	Fe I-CN	5²	5d	?	....	700
3824.46....	Fe I	6	15	6	2	1500
3825.86....	Fe I	20	20	8	3	1500
3827.79....	Fe I	8	10	3	1	1200
3829.35....	Mg I	10	40	30	14	6000
3830.68....	Fe I-CN	5⁴	5}			800
3831.06....	CN	3d	5}	1	....	800
3832.34....	Mg I	15	50	40	20	6000
3835.54....	H9	.....	100	90	50	7000
3836.69....	Zr II CN	3²	8	1	....	800
3838.30....	Mg I	25	60	50	25	7000
3840.44....	Fe I CN	8	10	4}		1200
3841.07....	Fe I-Mn I	10	10	4}	1	1200
3843.13....	Zr II-Fe I	10⁴	10d	0	....	800
3846.79....	CN	8⁴	8	0	....	750
3849.93....	Fe I	10	8	1	....	1000
3854.45....	CN-Fe I	4²	6}			800
3854.81....	CN	1	6}	0	....	800
3856.26....	Fe I-Si II	9²	20	10	3	1500
3858.36....	Ni I	7	6	0	....	750
3859.87....	Fe I	20	35	25	12	2500
3862.49....	Si II-CN	3²	5	0	....	900
3865.51....	Fe I	7	7	1	....	1000
3871.37....	CN	2d	8	0	....	750
3872.55....	Fe I	6	5	0	....	1000
3878.01....	Fe I-CN	8	8}			1500
3878.65....	Fe I-V II	11⁵	20}	8	2	1500
3883.29....	CN-Cr I	3²	15	*	....	750
3886.32....	Fe I-La II	15	25	12	2	1500
3889.20....	H8	.....	120	100	80	8500
3895.68....	Fe I	7	15	3	1	1200

WAVE-LENGTH	ELEMENT	R	0	600	1000	HT.
3899.70....	Fe I	8	10	2	?	1200
3900.54....	Ti II	5	40	15	6	2000
3902.97....	Fe I-Cr I	10	8}			1000
3903.10....	V II-Cr I	3²	6}	1	....	1000
3905.53....	Si I	12	12	0	....	800
3906.48....	Fe I	10	10	1	....	800
3913.55....	Ti II-Fe I	9²	40	20	8	2500
3920.25....	Fe I	10	15	4	1	1200
3922.92....	Fe I	12d	20	6	3	1500
3927.96....	Fe I	10²	20	*	....	1500
3930.26....	Fe I	8	20	7	1	1500
3933.90....	Ca II	1000	200	200	200	14000
3938.35....	Mg I-	6²	8	*	....	900
3944.03....	Al I	15	25	15	6	2000
3949.07....	La II-Ca I	1	8	?	....	750
3950.33....	V II	2	10	?	....	800
3951.97....	V II-Mn I	2	6	?	....	800
3958.23....	Zr II-Ti I	5	20	1	....	800
3961.51....	Al I	20	35	25	8	2000
3964.72....	He I	.....	8	*	....	1200
3968.70....	Ca II	700	175	175	175	14000
3970.25....	He	5	120	80	60	8500
3982.55....	Y II-Ti I	5²	12	1	....	900
4005.28....	Fe I	7	7}			1000
4005.74....	V II	3	8}	3	....	1000
4009.31....	He I	.....	2	2	....	1000
4012.41....	Ti II-Ce II	4	20	2	1	1500
4026.28....	He I	.....	30	30	25	5000
4028.36....	Ti II	4	12	1	....	800
4030.79....	Mn I	9²	20	8	....	1000
4033.07....	Mn I	7	18	6	....	1000
4034.47....	Mn I	6	15	5	....	1000
4045.84....	Fe I	30	30	12	2	1800
4063.62....	Fe I	20	20	8	1	1500
4071.75....	Fe I	15	15	6	1	1500
4077.83....	Sr II	8	80	60	30	6000
4101.85....	Hδ	40	140	120	100	8000
4120.78....	He I	.....	2	3	3	2500
4123.30....	La II	2	7	1	....	700
4132.13....	Fe I	10	8	1	....	900
4143.90....	{He I / Fe I}	..... / 15	10}	6	2	2000
4149.24....	{Zr II / Fe I}	2} / 4}	15d	1	....	900
4161.08....	Zr II-Fe I	4²	6	?	....	900
4163.66....	Ti II-Cr I	4	15	3	....	1000
4160.05....	He I	.....	1	?	....	1000
4171.95....	Ti II-Fe I	2	15	2	....	1000
4173.48....	Ti II-Fe II	8³	15	2	....	1000
4177.54....	Y II-Fe I	6²	20	2	....	1000
4178.87....	Fe II	3	15	2	....	1000
4187.77....	Fe I	10⁴	8	0	....	900
4191.48....	Fe I	9²	9d	0	....	800
4198.28....	Fe I	8²	6	0	....	800
4198.69....	Fe I-V I	3	2}			800
4199.09....	Fe I	5	6}	0	....	800
4202.11....	Fe I	8	10	1	....	900
4215.70....	Sr II	5	60	40	25	6000
4226.74....	Ca I	20	40	25	12	5000
4233.22....	Fe II	4	30	12	1	2200
4235.92....	Fe I-Y II	10³	10d	1	....	900
4246.90....	Sc II	5	50	35	20	5000
4250.16....	Fe I	8	9	0	....	900
4250.85....	Fe I-	9²	10	1	....	900
4254.36....	Cr I	8	25	12	2	1500
4260.51....	Fe I	10	10	5	....	1000
4271.16....	Fe I	6	6	1	....	800
4271.77....	Fe I	15	15	5	1	1500

* Ghost.

TABLE V—*Continued*

WAVE-LENGTH	ELEMENT	INTENSITY				HT.	WAVE-LENGTH	ELEMENT	INTENSITY				HT.
		R	o	600	1000				R	o	600	1000	
4274.77....	Cr I-Ti I	9²	20	10	2	1500	4464.56....	Ti II-Mn I	4²	7d	0	....	700
4287.96....	Ti II-Ni I	4³	6	0	....	700	4468.48....	Ti II	5	40	15	5	2500
4289.60....	Ca I-Cr I	9²	18	10)		1500	4471.54....	He I	....	80	80	70	7500
4290.18....	Ti II	2	18	10)	3	2000	4501.28....	Ti II	5	25	12	4	2500
4294.07....	Ti II Fe I	7²	18	12	3	2000	4508.32....	Fe II	4	12	2	....	900
4300.05....	Ti II	3	25	12	3	2000	4515.32....	Fe II	3	10	0	....	800
4301.94....	Ti II	2	8	2	?	1200	4520.22....	Fe II	3	12	1	....	800
4302.59....	Ca I-CH	8³	5	1	....	900	4522.67....	Fe II-Ti I	5²	18	2	....	1000
4303.20....	Fe II	2	4	1	....	900	4534.03....	Ti II-Fe II	7²	30	15	5	2500
4307.86....	Ti II-Ca II	9²	25	12	3	2000	4541.50....	Fe II-Cr I	3²	6d	0	....	700
4312.82....	Ti II	3	12	4	?	1200	4549.63....	Ti II-Fe II	8²	50	18	6	2500
4314.08....	Sc II	3	12	4	?	1200	4554.11....	Ba II	8	50	20	3	2000
4314.97....	Ti II-Fe I	8³	15	4	?	1200	4555.89....	Fe II	3	20	2	....	1000
4320.77....	Sc II-Ti II	5²	25	6	1	1500	4558.57....	Cr II La II	3²	15d	5	2	1500
4325.02....	Sc II	4	8	1	?	1200	4563.76....	Ti II	4	30	12	3	2500
4325.79....	Fe I-Ni I	10¹	15	8	2	1500	4572.00....	Ti II	6	35	15	5	2500
4337.98....	Ti II	4	15	6	1	2000	4583.86....	Fe II	4	25	6	0	1500
4340.63....	Hγ	20	160	140	120	8000	4588.20....	Cr II	3	8	2	....	1000
4351.84....	Fe II-Cr I	10²	15	4	0	1200	4589.93....	Ti II	3	10	1	....	1000
4374.49....	Sc II-Fe I	3	10	3	....	1000	4618.81....	Cr II-Fe I	4	6	0	....	800
4375.00....	Y II-Mn I	2	12	4	....	1000	4629.42....	Fe II-Ti I	6	20	2	....	1000
4376.00....	Fe I	6	8	3	....	800	4634.04....	Cr II	2	6	0	....	800
4383.54....	Fe I	15	15	10	2	1600	4685.83....	He II	....	2 N	3	2	3500
4387.86....	He I	....	2	4	3	2000	4713.15....	He I	....	5	8	5	5000
4395.13....	Ti II-V I	5²	40	15	5	2500	4805.13....	Ti II	3	15	0	....	800
4399.79....	Ti II	3	10	1	....	800	4823.46....	Mn I-Y II	5	10	0	....	750
4400.52....	Sc II-Ti II	4²	9	1	....	800	4824.09....	Cr II-Fe I	3	12	0	....	750
4404.79....	Fe I	10	12	4	0	1200	4861.50....	Hβ	30	200	180	160	8500
4415.12....	Fe I	8	8	0	....	800	4922.01....	He I	....	3	4	3	2500
4415.53....	Sc II	3	8	1	....	800	4923.96....	Fe II	5	30	6	0	2000
4416.84....	Fe II	2	6	0	....	800	4934.08....	Ba II	7	25	3	0	1200
4417.71....	Ti II	3	12	3	?	1200	4957.30....	Fe I	5	4)	0	....	600
4427.34....	Fe I	5	8	2	....	800	4957.58....	Fe I	8	8)		....	600
4434.98....	Ca I-Fe I	7²	7	1	....	800	5015.68....	He I	....	2	4	1	2500
4437.70....	He I / V I	.... / 1²	2d	0	....	750	5018.44....	Fe II	4	25	5	0	2000
4443.85....	Ti II	5	30	15	5	2500	5167.35....	Mg I / Fe I	15) / 5)	18	3	0	1500
4450.37....	Ti II-Fe I	3²	9	3	....	1000	5168.99....	Fe II-Fe I	7²	25	2	....	1500
4454.77....	Ca I-Zr II	5	7	2	....	800	5172.65....	Mg I	20	30	10	1	2000
4461.60....	Fe I	4	5	1	....	700	5183.58....	Mg I	30	40	12	2	2500

and 1500 km. This is in agreement with the findings of various other observers.[10] The density gradient above 1000 km is smaller than that of any other element observed, the density gradient of *He* II being even less than that of *He* I. This we derive from the fact that the *He* I lines of a given I_{1000} rise to much greater heights than lines of any other element with the same I_{1000}.

Hydrogen.—It is very difficult to compare I_R and I_o for hydrogen because of the unique broad and shallow character of the *H* lines in the Fraunhofer spectrum. However, if we accept Unsöld's revised figures[11] for the number of atoms contributing to each of the absorp-

[10] Dyson, *Philosophical Transactions*, A, **206**, 449, 1906; Davidson and Stratton, *Memoirs of the Royal Astronomical Society*, **64**, 138, 1927; Pannekoek and Minnaert, *op. cit.*

[11] *Zeitschrift für Physik*, **59**, 364, 1930.

tion hydrogen lines and convert this number into the corresponding Rowland intensity by means of Russell's tables,[12] we obtain a Rowland intensity of 3; in other words, if the H lines were not broadened by the Stark effect, the Rowland intensities of the first eight or ten members of the Balmer series would average 3. It would follow that the density gradient of atoms emitting the Balmer series is very small between the levels represented by I_R and I_0. Between the levels of I_0 and I_{600}, the decrease in density is about one half as great as in the case of Ti II. Above the level represented by I_{600}, there is very little difference between the density gradient of H and that of Ti II.

Ionized elements.—The behavior of the ionized elements in general is similar to that of Ti II; hence it seemed legitimate, in the preceding section, to group Cr II and Sc II with Ti II in deriving the density distribution. The Ti II lines $\lambda\lambda$ 3685, 3759, and 3761, all of which drop to the lowest doublet level, a^2F of excitation potential 0.58, seem to show a smaller density gradient than other Ti II lines of both higher and lower excitation potential. These are the most intense lines in the Ti II spectrum, in the laboratory, in the reversing layer, and in the flash. It is difficult to establish the smaller density gradient definitely because of the great difference in intensity and because so few lines are involved. In general, the decrease in density of ionized atoms between the levels of I_R and I_0 is much less than the decrease in density of neutral atoms between these two levels. Above the level of I_0, the density distribution of the ionized elements is not very different from that of the lines of low excitation potential of neutral elements.

The behavior of the ultimate doublet of the ionized alkaline earths is of interest. The rate of decrease in density for the doublet of Ba II is rather greater than for Ti II, whereas for the Sr II doublet it is probably somewhat less than for Ti II. The H and K lines of Ca II are so much more intense than any other lines that a comparison is impossible. The Mg II ultimate doublet is inaccessible at λ 2800.

Neutral elements.—The ultimate lines of certain neutral elements show a markedly smaller density gradient, in general, than the aver-

[12] *Astrophysical Journal*, **70**, 11, 1929.

age line of the same element, although a few exceptions exist. Attention has already been called to these,[13] in particular to the ultimate Fe I multiplet $a^5D - z^5D^o$ and other iron multiplets. The $a^7S - z^7P^o$ ultimate multiplet of Cr I at $\lambda\lambda$ 4250–4300 is perhaps even more striking; the decrease in density of atoms producing these lines, between the level of I_R and that of I_0, is only about one-tenth of the decrease in density of neutral atoms in general;[14] again, the three lines of this multiplet are of average intensity 7 in Rowland and are visible to 1500 km in the flash, whereas an average neutral line of Rowland intensity 7 in this part of the spectrum is visible to 800 km only. The ultimate multiplet $a^7S - y^7P^o$ of Cr I does not show nearly such a marked effect, however. λ 4227 of Ca I, the $a^6S - z^6P^o$ multiplet of Mn I at $\lambda\lambda$ 4030–4035, and the $a^2P^o - z^2S$ multiplet of Al I at $\lambda\lambda$ 3940–3965, all show the effect to a greater or less extent, all being ultimate lines. λ 4227, of Rowland intensity 20, is visible to 5000 km, whereas a subordinate line of a neutral atom of the same Rowland intensity in the same part of the spectrum would be visible to, perhaps, 1500–1800 km.

Most striking of all, however, is the behavior of the Mg I subordinate multiplet $z^3P^o - e^3D$ of excitation potential 2.70. Whereas, in general, subordinate lines of similar intensities in Rowland would be visible to 1000, 1300, and 1600 km, respectively, the Mg I lines are visible to 6000, 6000, and 7000 km, respectively. These are approximately the heights to which the ionized elements would be visible. But the most interesting point is that the density gradient above the level of I_{1000} is less than for any other element observed, except helium. λ 4227 of Ca I also shows a smaller density gradient than the average above the level of I_{1000}. Thus it is probable that above 1000 km, Mg II and Ca II are largely supported by radiation pressure in the manner suggested by Milne. It seems safe to conjecture, then, that the ultimate doublet of Mg II at λ 2800 must extend much higher than the H and K lines, not only on account of the relative behavior of the lines of Mg I and Ca I, but also on account of the greater abundance of Mg than of Ca on the sun.[15] Hence, of

[13] St. John, *loc. cit.;* Mitchell, *op. cit.*, 157, 1930, see Table VI.

[14] Estimated from the calibration of the I_0's explained in sec. III.

[15] Russell, *loc. cit.*

course, an upper chromosphere composed chiefly of Ca II is really non-existent, as has already been pointed out by others.[16] It should, perhaps, be mentioned explicitly that the subordinate lines of Ca I apparently behave similarly to the subordinate lines of other neutral elements, as do the lines of Mg I of excitation potential greater than 2.70.

III. RELATIVE ABUNDANCE FROM INTENSITY ESTIMATES

A method of estimating the relative abundance of elements similar to that used by Russell[17] would be satisfactory provided: (a) self-reversal can be properly allowed for; (b) all elements have the same density distribution in the chromosphere; (c) the distribution of atoms of a given element in various stages of excitation and ionization approximates the distribution in thermodynamical equilibrium, or deviates from it in a predictable manner; and (d) the atomic absorption coefficient is independent of the excitation potential.

Concerning (a), we have assumed that all lines of any given intensity in a limited spectral region are affected to the same extent by self-reversal.

As for (b), it is obvious that all elements do not have the same density distribution, but, as we have found in the preceding section, above the level corresponding to I_0 there is little difference between the distribution of ionized elements and that of the atoms emitting the ultimate lines of the more abundant neutral elements (H, He, Mg, and Ca excepted). However, since the great majority of atoms of any element are in states of low excitation, we are not making a very wild assumption in treating the neutral elements as a whole as if they had the same density distribution as the ionized elements above the level I_0.

Concerning (c), it is not to be expected that matter and radiation in the chromosphere are in thermodynamical equilibrium. However, Pannekoek[18] has derived the correction to be applied to Saha's ionization equation for conditions similar to those in the chromosphere where the temperature of the radiation is higher than that

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[16] Russell, Dugan, and Stewart, *Astronomy*, p. 578, 1927; and see also Unsöld, *loc. cit.*, concerning the probability of a preponderance of hydrogen.

[17] *Loc. cit.*

[18] *Bulletin of the Astronomical Institutes of the Netherlands*, **3**, 207, 1926.

corresponding to the translatory motions of the molecules. Furthermore, since the pressures are so low in the chromosphere, we have assumed that excitation by collision is negligible compared with excitation by radiation.

We have no satisfactory knowledge about the point raised in (d), and this must be kept in mind in interpreting the results of this section.[19]

The calculations may be divided into three steps: first, the calibration of the intensity scale in terms of the relative numbers of emitting atoms for the various regions of the spectrum; second, the determination of the relation between the intensity scale in one region of the spectrum and the scale in any other region; and, third, the derivation of the relative abundance of various types of atoms.

Calibration of intensity scale in any particular spectral region.—Ninety-nine multiplets involving about five hundred lines were available for calibrating the intensity scale (I_0 scale) in terms of relative numbers of emitting atoms. This is four-tenths as many as were available to Adams, Russell, and Moore in calibrating the Rowland scale. The relation between I_0 and the relative number of atoms, N, for any particular region is the same for all regions to the violet of λ 5200. It is given in Table VI, and is very close to $I_0 = 1.06\ N^1$, for values of I_0 greater than 4. The residuals indicate that the probable error in the determination of log N from a single line in any given region of the spectrum is about 0.3. That this is no larger than that found by Adams, Russell, and Moore is accounted for, we believe, by the somewhat more accurate and homogeneous intensity estimates in the I_0 series than in the I_R series.

TABLE VI

I_0	1	2	3	4	5	6	7	8	9	10	12	15	18	20	25	30
Relative number of atoms, N	5	8	12	17	24	33	44	56	69	83	115	170	250	320	560	800
log N	.70	.91	1.08	1.23	1.38	1.52	1.64	1.75	1.84	1.92	2.00	2.24	2.40	2.51	2.75	2.90

Relative calibration of various spectral regions.—In order to relate the intensity scale in one region of the spectrum with that of another, Russell used Unsöld's measures of the total energy absorption of ten

[19] On this point see Menzel, *op. cit.*, p. 276, 1931.

of the Fraunhofer lines in the region from λ 3944 to λ 5896. In our case, the only measures of total emission available were those of Pannekoek and Minnaert[20] in a narrow region of the spectrum from λ 4154 to λ 4768. Hence, we have used instead the transition probability formulae of Kronig[21] for multiplets in certain triads. These formulae have been tested by Harrison and Engwicht,[22] using the emission spectrum of Ti. In order to compare the observed intensi-
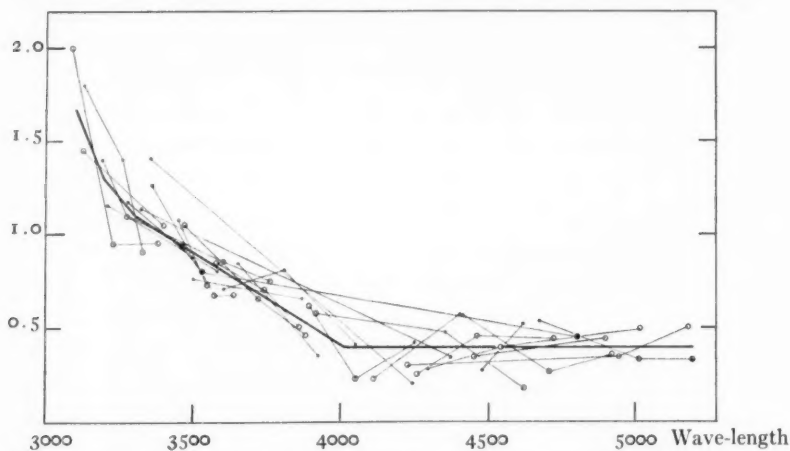
Fig. 4.—Calibration-curve for flash spectrum between λ 3066 and λ 5200. The ordinates give the logarithms of the relative numbers of atoms producing a line of given intensity. The large circles represent well-determined and the small circles poorly-determined points. The three large filled circles refer to the triad of the illustrative example.

ties with the predicted, it was necessary to multiply the Kronig transition probabilities by $e^{-\frac{h\nu}{kT}} \times \nu^4$, where ν is the frequency of the multiplet and $e^{-\frac{h\nu}{kT}}$ is the usual Boltzmann expression for the excitation function; the second factor, referred to as the ν^4 correction, "arises from the expression for the intensity of radiation from a classical oscillator in terms of frequency," to quote Harrison and Engwicht. A very fair agreement is found between predicted and observed intensities, even in the extreme case where the individual multiplets in a triad are separated by 800 A.

[20] Op. cit.   [21] Zeitschrift für Physik, **33**, 261, 1925.
[22] Journal of the Optical Society of America, **18**, 287, 1928.

In our calibration we have used the triads of the elements Ca to Ni of the iron group for which electron configurations have been assigned by Russell.[23] Twenty-six triads involving sixty-nine multiplets were available, giving a calibration of the scale from λ 3066 to λ 5200. An idea of the degree of accordance of the data is given by Figure 4, in which each group of three points connected by lines represents a triad. The abscissae are angstrom units; the ordinates

TABLE VII

Multiplet (1)	Wave-Length (2)	Estimated Flash Intensity I_0 (3)	log N from Table VI (4)	Kronig Relative Transition Probability F (5)	log F (6)	$\log e^{-\frac{h\nu}{kT}} \cdot \nu$ minus Constant (7)	log Theoretical Relative Number of Atoms (8)	Col. 8 minus Col 4 (9)	Adopted Mean Correction to Col. 4 (10)
b²P−y²P⁰......	3443.4	3*	1.08:	33	1.52	.70	2.22	1.14:	1.10
	3452.4	5	1.38	67	1.83		2.53	1.15	
	3456.4	10	1.92	167	2.22		2.92	1.00	
	3465.6	10†	.....	33	1.52		2.22	.....	
				300					
b²P−x²D⁰......	3520.2	10	1.92	167	2.22	.73	2.95	1.03	.95
	3533.8	4	1.23	33	1.52		2.25	1.02	
	3535.4	20	2.51	300	2.48		3.21	.70	
				500					
b²P−z²S⁰......	4780.0	10*	1.92:	33	1.52	1.01	2.53	.61:	.60
	4805.1	15	2.24	67	1.83		2.84	.60	
				100					

* Slightly blended.
† Heavily blended.

are logarithms of the relative numbers of atoms contributing to a line of given estimated intensity.

As an example of the method, consider for the purpose of this illustration the three points denoted by filled circles. These represent the Ti II triad (b²P−y²P⁰), (b²P−x²D⁰), and (b²P−z²S⁰). The parent-term of all these terms is a³P of the Ti III spectrum. The triad represents the electron jump (3d)²4s − (3d)²4p. The details are given in Table VII, the first four columns of which are self-explanatory. Column 5 gives the Kronig relative transition probabilities.

[23] *Astrophysical Journal*, **66,** 184, 283, 347, 1927.

Thus the transition probabilities of the three multiplets of the triad are in proportions of 300, 500, and 100, respectively, and, similarly, the transition probabilities of the most intense lines in each of the multiplets are in proportions of 167, 300, and 67, respectively. Column 6 gives the log of the items in column 5. In column 7 is the log of the appropriate modifying factor, to obtain the number of atoms to be expected when the exciting radiation is of temperature 5740°. The items in column 8 are the sums of the items in columns 6 and 7, and represent the logs of the theoretical relative numbers of atoms producing each of the lines. By comparing these with column 4, we derive the corrections given in column 10, which should be added to the items in column 4. Thus, from this data we find that the calibration-curve at λ 3450 should be 0.15 higher than at λ 3530, and 0.50 higher than at λ 4790.

Derivation of relative abundances.—As mentioned above, we have used Pannekoek's modification of Saha's ionization equation. Following Russell's notation, in the main, we have the equation:

$$\log \frac{A_1}{A_0} = \log \frac{S_1}{S_0} - \log \frac{B'(T_{\text{eff}})}{B(T_{\text{eff}})} = -\frac{5040}{T_{\text{eff}}} (I - I_0) , \qquad (15)$$

where A_0 is the abundance factor for neutral atoms; A_1 is the abundance factor for ionized atoms; S_0 is the total number of neutral atoms per unit volume; S_1 is the total number of ionized atoms per unit volume; $B(T_{\text{eff}})$ is the partition function for neutral atoms at temperature T_{eff}; $B'(T_{\text{eff}})$ is the partition function for ionized atoms at temperature T_{eff}; T_{eff} is the effective temperature of the radiation, assumed to be 5740°; I is the ionization potential of the neutral atom; and I_0 is the "level of ionization" of the chromosphere. The equation which connects the level of ionization, I_0, with the temperature and electron pressure of the chromosphere follows:

$$I_0 = \frac{T_{\text{eff}}}{5040} (\log T_{\text{eff}} + \tfrac{3}{2} \log T - 6.5 - \log P_e) , \qquad (16)$$

where T is the temperature corresponding to the translatory energy of the electrons in the chromosphere and P_e is the electron pressure in the chromosphere expressed in atmospheres. We do not need this

equation, however, to derive the relative abundances for we can determine I_0 observationally. Thus, the relative abundances do not depend on any assumption as to the temperature of the chromosphere. Finally, neglecting excitation by collisions, we have

$$\log A = \log M - \log w + \frac{5040E}{T_{\text{eff}}}, \qquad (17)$$

where M is the number of atoms which are in any given state of excitation; w is the total quantum weight of the state; and E is the excitation potential of the state.

For a detailed explanation of the procedure, the reader is referred to Russell's original paper. We have followed him, except that we have made no empirical adjustment for departures from thermodynamical equilibrium, since equations (15)–(17) are supposed to allow for this. The results are given in Table VIII. The first three columns are self-explanatory. In the column headed "Lines" we have given the number of lines on which the estimate of log M chiefly depends, rather than the total number of lines of that excitation potential appearing in the spectrum. In the simpler spectra, frequently three or four strong lines involve 90 per cent of the atoms at a given excitation potential, although ten or fifteen fainter lines also may appear. Hence the error in the estimate of log M depends almost entirely on the errors of the estimated intensities of the few strong lines, and we think it a better index of the weight of log M to give the number of the principal lines. In complex spectra, on the other hand, we have used all the unblended lines of each of the stronger multiplets to determine the numbers of atoms contributing to the multiplet as a whole. We may have five or ten determinations of this for one multiplet. The average value is taken, so that the fainter lines in this case have equal weight with the brighter lines of the multiplet. However, the lines of the unimportant multiplets have not been counted. We have explained this in full lest it be thought that our estimates rest on very much less data than others. When a principal line or multiplet was too strong for the calibration-curve, or to the red of the calibrated region, we have made a rough estimate and appended the letter s or r to the number of lines.

TABLE VIII

El.	Term	E.P.	Lines	log M	log A	Mean log A	I	I_0	Comp log A
H	²S, ²P	10.15	18s	$\rangle\rangle$4.2:	$\rangle\rangle$12.3:	$\rangle\rangle$12.	13.54	. . . . .	. . . . . .
He I	¹S	20.52	1	$\rangle$1.2	$\rangle$18.9				
	³P	20.87	4s	$\rangle$3.0	$\rangle$20.3:	$\rangle$20.			
	¹P	21.13	2	$\rangle$1.6	$\rangle$19.6		24.48	49::	. . . . . .
He II	²S	48.16	1	$\rangle$0.2	$\rangle$42.2	$\rangle$42.			
Be II	²S	0.00	1	1.7:	1.4:	1.4:	9.29	. . . . . .	
Na I	²S	0.00	2r	2.1	1.8:	1.8	5.11	. . . . . .	
	²P	2.09	4r	$\rangle$0.6	$\rangle$1.6				
Mg I	³P	2.70	6s	3.4:	4.8:	5.0			5.3
	¹P	4.33	3	$\rangle$1.5	$\rangle$4.8		7.61	10.2:	
Mg II	²D	8.83	1	0.6	7.3:	7.3:			6.6
Al I	²P	0.00	2	$\rangle$2.6	$\rangle$1.8	$\rangle$1.8	5.95	. . . . . .	
Si I	¹S	1.90	2	1.6	3.3	3.3:			5.2:
Si II	²D	6.83	1	$\rangle$0.9	$\rangle$5.9		8.14	13.5:	
	²S	8.09	2r	1.1:	7.9:	8.0:			6.1:
	²P	10.03	1r	0.2:	8.2:				
Ca I	¹S	0.00	1s	2.3:	2.0:	2.4			2.3
	³P	1.88	3r	$\rangle$1.8:	$\rangle$2.5:				
	³D	2.51	4r	1.4:	2.4:		6.09	8.8:	
Ca II	²S	0.00	2s	$\rangle\rangle$2.5	$\rangle\rangle$2.2	4.8			4.9
	²P	3.12	4	2.7	4.8				
Sc I	²D	0.00	3	1.1	$-$0.2	$-$0.2			$-$0.2
Sc II	³D	0.00	12	3.1	1.9		6.57	9.1	
	¹D	0.31	2s	2.5:	2.1	2.0			2.0
	³F	0.61	4	$\rangle$2.1	$\rangle$1.3				
Ti I	³F	0.02	10	2.1:	0.5:				
	⁵F	0.82	14	1.8	1.0	0.9			0.9
	³G	1.87	3	1.1	1.3				
Ti II	⁴F	0.02	19	3.9	2.5				
	⁴F	0.13	20	$\rangle$3.3	$\rangle$2.0				
	²F	0.58	5s	3.5:	2.9:				
	²D	1.08	10	2.9	2.8				
	²G	1.12	8s	$\rangle$2.9	2.6		6.80	9.1	
	⁴P	1.16	7	$\rangle$2.4	$\rangle$2.3				
	²P	1.22	12	2.7	3.0	2.9			2.9
	⁴P	1.23	11	2.8	2.8				
	²D	1.56	12	2.6	3.0				
	²H	1.57	4	2.8	2.8				
	²G	1.88	8	2.7	3.1				
	²P	2.04	9	2.5	3.5				
	²F	2.58	5	$\rangle$1.7	$\rangle$2.8				

TABLE VIII—*Continued*

El.	Term	E.P.	Lines	log M	log A	Mean log A	I	$I_\circ$	Comp. log A
V I.....	^{4}F	0.03	11	)1.3	)−0.4	0.5			0.4
	^{6}D	0.28	8	1.7	0.5				
							6.76	8.8	
V II....	^{5}F	0.36	4	)2.4	) 1.2	2.3			2.4
	^{3}F	1.09	10	2.6	2.3				
	^{3}G	1.80	3	)1.3	) 1.7				
Cr I.....	^{7}S	0.00	4	2.6	1.8	1.9			2.3
	^{5}S	0.94	4	)1.7	) 1.8				
	^{5}D	0.98	14	)1.9	) 1.1				
	^{5}G	2.53	3	1.2	1.8		6.74	)9.7	
Cr II....	^{4}D	2.43	18	)3.7	) 4.5	) 4.5			4.4
	^{4}P	2.69	3	))2.1	)) 3.4				
Mn I....	^{6}S	0.00	3	)2.2	) 1.1	2.5			2.5
	^{6}D	2.14	9	2.0	2.4				
	^{8}P	2.29	6	1.7:	2.3:		7.40	))8.0	
	^{4}D	2.91	10	1.5:	2.8:				
Mn II...	^{5}D	1.83	6	))2.8	))3.0	)) 3.0			4.0
Fe I....	^{5}D	0.07	19	)3.3	) 1.7	2.8			3.0
	^{5}F	0.99	37	3.2	2.5				
	^{3}F	1.54	17	2.8	2.8				
	^{5}P	2.19	32	2.3	3.0				
	^{7}D	2.41	9	2.1:	2.7:				
	^{7}F	2.84	19	2.2	3.0		7.83	))8.7	
Fe II....	^{4}D	1.04	5	))2.5	)) 2.1	)) 3.6			4.1
	^{4}P	1.69	8	))2.9	)) 3.3				
	^{4}P	2.68	8	))2.2	)) 3.5				
	^{4}F	2.82	15	))2.6	)) 3.6				
Co I.....	^{4}F	0.12	8	)1.9	) 0.3	1.2	7.81	....	
	^{4}F	0.54	6	2.1	1.1				
	^{2}F	0.98	6	1.6	1.3				
Ni I	^{3}D	0.12	9	2.6	1.5	1.6			1.8
	^{3}F	0.14	4	)2.0	) 0.5				
	^{1}D	0.42	7	1.9	1.6		7.64	)9.1	
Ni II....	^{2}D	2.89	2	)1.4	) 2.9	) 2.9			3.1
	^{2}G	4.02	1	)0.4	) 2.7				
Cu I....	^{2}S	0.00	2	1.6	1.3	1.3	7.69	....	
Zn I....	^{3}P	4.02	3	1.1	3.7	3.7	9.36	....	
Sr I.....	^{1}S	0.00	1	0.2	0.2	0.2			−0.1
							5.65	8.6:	
Sr II....	^{2}S	0.00	2S	3.0:	2.7:	2.8			2.9
	^{2}P	2.97	2	1.0	2.8				

TABLE VIII—*Continued*

El.	Term	E.P.	Lines	log M	log A	Mean log A	I	I_0	Comp. log A
Y II....	^{1}S	0.00	3	$\rangle$1.4	$\rangle$1.1				
	^{3}D	0.10	9	2.8	1.7	1.7	6.5	....	
	^{1}D	0.40	3	2.0	1.7				
	^{3}F	1.00	9	2.0	1.6				
Zr II....	^{4}F	0.08	12	2.7	1.3				
	^{2}D	0.54	4	2.2:	1.7:	1.4	6.92	....	
	^{2}F	0.75	9	1.9	1.4				
Ba II....	^{2}S	0.00	2	2.6	2.3				
	^{2}D	0.65	31	$\rangle$2.2	$\rangle$1.7	2.4	5.19	....	
	^{2}P	2.60	3	1.1	2.6				
La II....	^{3}F	0.12	5	1.5	0.3				
	^{1}D	0.17	4	1.6	1.0	0.7	5.5	....	
	^{3}D	0.32	5	1.8	0.9				

The value of log A which is given in the sixth column is calculated by means of equation (17) from the value of log M given in the fifth column. From these values of log A we can form an idea of the inter-agreement of the data, since all the values of log A within a group should, of course, be equal. It appears that for excitation potentials less than 3 volts there is no very certain dependence of log A on excitation potential. Furthermore, it is difficult to disentangle such a dependence from errors in calibration, since usually the lines of high excitation potential are weaker than, and lie to the red of, lines of low excitation potential. In the seventh column is given a weighted mean of the values of log A in the sixth column. In the eighth column is the ionization potential.

In the ninth column is given the level of ionization calculated from equation (15) wherever possible. Ideally, all values of I_0 should be the same. Obviously, the ionization theory cannot account for the behavior of He, as has long been known. Si, too, gives a very discordant value of I_0. Since in both cases the computation of I_0 depends on lines of unusually high excitation potential, we might ascribe the discordance to an unknown excitation-potential effect. On the other hand, the value of I_0 obtained from Mg is not particularly discordant, and a high excitation potential is involved here too. As mentioned above, we have no knowledge that the atomic ab-

sorption coefficient is the same for all atoms of all excitation potentials. If we disregard the values of I_0 obtained from He and Si, and assign a weight of 2 to the values from Sc, Ti, and V, and unit weight to the values from Mg, Ca, and Sr, we obtain for I_0 the value 9.1 ± 0.1 volts. If we add 0.5 to the lower limits for I_0 obtained from Cr and Ni and give each of these a weight $\frac{1}{2}$, the value of I_0 becomes 9.2 ± 0.1; thus it is very little changed. We have adopted the value 9.1 and computed with it values of $\log A_0$ and $\log A_1$, which give the best accordance with the data. These are given in column 10.

From the determination of I_0 we may deduce the electron pressure near the base of the chromosphere by means of equation (16). If we assume that the temperature there is equal to the theoretical boundary temperature of the sun, $2^{-\frac{1}{4}} \times T_{\text{eff}}$, or $4800°$, we find P_e equal to 6×10^{-6}, a value somewhat greater than that which is usually quoted.

Following Russell's procedure again, we have computed $\log S_0$ and $\log S_1$, and from these $\log T$ and $\log Q$, T being proportional to the total number per cm³ of neutral and ionized atoms of any element in the lower chromosphere and Q being proportional to the total weight of these atoms. Values of $\log T$ and $\log Q$ are given in Table IX, arranged in order of increasing atomic numbers, together with a comparison with Russell's values for the reversing layer, Miss Payne's for stellar atmospheres in general,[24] Clarke and Washington's for the earth's crust,[25] and Merrill's values for stony meteorites.[26] In the last column is a comparison with Menzel's results from the chromosphere, which are further discussed in the next section.

The comparison with the reversing layer, which is the lower and denser part of the chromosphere, shows fairly conspicuous differences which, at first glance, might be interpreted as real. However, investigation shows that they may very well be attributed to differences between Russell's calibration and ours. Since Russell's calibration-curve to the violet of λ 3900 is extrapolated, the present material forms an independent check on it. As regards hydrogen, the

[24] Proceedings of the National Academy of Sciences, 11, 192, 1925; Harvard Observatory Bulletin, No. 835, 1926.

[25] Proceedings of the National Academy of Sciences, 8, 114, 1922.

[26] Ibid., 1, 429, 1915.

differences between the chromosphere and the reversing layer were formed from the values of log T given in Table XIV of Russell's paper. Subsequently in that paper and in a later paper[27] he adopts a smaller value as more probable. This is indicated in the footnote to Table IX.

TABLE IX

ELEMENT	LOG T	LOG Q	CHROMOSPHERE *minus*				COL. 2 *minus* MENZEL
			Reversing Layer	Stellar Atmospheres	Earth's Crust	Stony Meteorites	
H	»12.	»12.	»+1. *	» 0	»+4.	»+6.	»−1.
Be	2.2:	3.2:	+1.1:	. . . .	. . . .	. . . .	. . . .
Na	5.6:	7.0:	− .9:	− .6:	− .7:	+ .4:	− .9:
Mg	6.9	8.3	− .2	+ .3	+ .7	+ .4	+ .2
Al	) 4.9	) 6.3	)− .8	)−1.1	)−1.9	)− .7	)− .6
Si	7.0:	8.4:	+ .4:	+ .4:	− .3:	+ .3:	+2.0:
Ca	5.3	6.9	− .7	− .5	− .9	0.0	− .6
Sc	3.4	5.1	+ .5	. . . .	+3. :		+ .3
Ti	4.7	6.4	+ .2	− .4	− .7	+ .6	− .3
V	4.0	5.7	− .3	0.0	− .2	. . . .	+ .1†
Cr	5.3	7.0	+ .3	+ .4	+ .9	+ .7	− .1
Mn	4.9	6.6	− .3	− .7	+ .3	+ .5	− .8
Fe	5.8	7.5	− .7	0.0	− .5	− .7	− .5
Co	3.9	5.7	−1.0	. . . .	+ .9	− .2	− .4
Ni	4.2	6.0	−1.1	. . . .	+ .2	−1.0	−1.1
Cu	2.8:	4.6:	−1.5:	. . . .	− .7:	− .4:	. . . .
Zn	4.2:	6.0:	0.0:	−1.0:	+1.1:	. . . .	+ .6:
Sr	3.2	5.1	+ .6	+ .6	. . . .	. . . .	+ .8
Y	3.0	4.9	+1.1	. . . .	. . . .	. . . .	+ .2
Zr	3.1	5.1	+1.3	. . . .	. . . .	. . . .	+ .7
Ba	3.1	5.2	+ .5	+1.0	. . . .	. . . .	+ .1†
La	2.2	4.3	+1.1	. . . .	. . . .	. . . .	+ .4†

* This would be »+3, if Russell's revised estimate be taken.

† There are minor errors in Menzel's values of log T for V, Ba, and La. These have been corrected in deriving the foregoing differences.

One of the most significant features of Table IX is that the average difference between two independent determinations of the composition of the chromosphere, as given in the last column, is as large as is found in the comparisons of the composition of the chromosphere with that of other samples of matter. From this we infer that it is impossible to draw any more definite conclusion than that within the errors of observation the composition of all the samples is alike, with the exception that H is conspicuously deficient in the earth's crust and in meteorites, and is probably more abundant in

[27] *Op. cit.*, **75**, 337, 1932.

the chromosphere than in the other samples. It also seems safe to conclude that there is no marked deficiency of the heavier elements in the chromosphere. This indicates, again, that the elements are kept very well stirred up by some agency which may well be turbulence.

IV. NOTES ON MENZEL'S STUDY OF THE CHROMOSPHERE

In the second publication from this observatory dealing with *Fe* I lines in the flash spectrum, it was found that both heights and intensities have an intimate connection with excitation potentials. For instance, a line of intensity 6 in the sun extends to 1000 km if the line originates from a low excitation potential but to only 400 km if the excitation potential is high. Since the *Fe* I lines of heights 1000 km and greater are found mainly in multiplets of small excitation potential, while heights of 300 or 400 km belong to multiplets of all excitation potentials, the conclusion was drawn that "so long as spectral observations on the sun are grouped together on Rowland's or any other scale, the average can represent only a first approximation to the truth." Hence in the present discussion we have attempted to group together only those lines produced under approximately the same conditions.

Menzel's methods are very different. He had to combine measures made with four different spectrographs. Starting with the 1905 moving-plate spectrum, he derives from 56 lines a relation between estimated flash intensities and theoretical intensities by means of the multiplet-intensity formulae. He treats the 1905 ultra-violet spectrum in the same way. To relate the latter to the former, he assumes that all lines which extend to a particular height should have the same "theoretical intensity," an assumption which we would question in view of the above-mentioned evidence and of that in sections I and II of this paper. But the heights from the two plates are not strictly comparable because of the different sensitivity of the instruments, length of exposure, etc., which cause zero-point and scale corrections which will be discussed in a later paper. To correct for the difference in the speed of the instruments he introduces a factor of 300, i.e., a correction of 2.5 to the log of the theoretical intensity. To correct for zero-point he assumes that the ultra-violet

heights are 500 km too low. Incidentally, this is done in spite of the fact that in correlating the heights from the various plates he finds that the 1905 fixed plate and the 1908 moving plate also give heights about 500 km lower than the 1905 moving plate; thus the heights from three plates, which agree very well among themselves, are assumed to be systematically in error because they disagree with the heights from a fourth plate.

We wish to call attention to some few details where his discussion and ours give conflicting results.

Temperature determinations.—In his § 18, Menzel investigates the dependence of flash intensities on excitation potential for *Fe* I. In his Figure 40, he has plotted log N_a/N_a' against excitation potential, where N_a/N_a' is the relative number of atoms in the flash spectrum as compared with the number in the solar absorption spectrum. Figure 40 is rather misleading on account of the fact that the Russell calibration and the Menzel calibration are not parallel for lines of different intensity. Thus, from examining nine of the stronger multiplets in Menzel's data, we find that the value of log N_a/N_a' for the lines of medium intensity (Rowland intensity 5–8) in a given multiplet exceeds by 0.26 ($\pm$0.05) its value for the more intense lines in the same multiplet. Similarly, in eighteen multiplets of medium intensity, we find the value of log N_a/N_a' for lines of Rowland intensity less than 5, exceeds by 0.42 ($\pm$0.05) its value for lines of Rowland intensity 5–8 in the same multiplet. Now the points on the left of Menzel's diagram represent, on the average, much more intense lines than the points on the right of the diagram. It follows that the point of 0.0 E.P. should be raised by perhaps 0.3 units relative to the point at 2 volts, and the point at 3.8 volts should be depressed by some such amount. Further evidence that Menzel's curve in Figure 40 slopes in the wrong direction between 2 and 4 volts is to be found on page 10 of this paper. The analysis there given of the dependence of the density gradients of *Fe* on excitation potential indicates that at 3 volts it is more probable that the density gradient is increasing rather than decreasing with increasing excitation potential. This would indicate that the slope of the curve in Menzel's Figure 40 should be minus at 3.0 volts instead of plus. The analysis just referred to is independent of all calibrations of

intensities, and the sign of the slope would not be altered even if the observed heights were systematically in error.

These corrections to Figure 40 would lower the value of the temperature of the chromosphere derived by Menzel from about 4400° to about 3800°. It seems to us, however, in view of the observed deviations from thermodynamical equilibrium, that such determinations of the temperature of the chromosphere are of little value. Thus, if we treat the data in Table II of this paper in a manner similar to that used by Davidson, Minnaert, Ornstein, and Stratton[28] and later by Menzel in his § 18, we may derive temperature gradients in the chromosphere as follows: Assuming thermodynamical equilibrium (which is quite unjustified, it seems to us), we have, by arguments similar to those used by the foregoing authors:

$$\frac{1}{T_{h+\Delta h}} - \frac{1}{T_h} = \frac{1}{5040} \frac{d}{d\text{E.P.}} \left(\log \frac{E_h}{E_{h+\Delta h}} \right).$$

There is an average difference of 1 volt between the excitation potential of the data in column 4 of Table II and that in column 5, whence

$$\frac{1}{T_{1000}} - \frac{1}{T_{800}} = \frac{1}{5040} \frac{.39 - .17}{1.0} = .00004 \pm .00001,$$

and similarly

$$\frac{1}{T_{1200}} - \frac{1}{T_{1000}} = .00004 \pm .00001$$

and

$$\frac{1}{T_{1500}} - \frac{1}{T_{1200}} = .00004 \pm .00001.$$

The probable errors given above are derived from those in Table II, which, in turn, were derived from the interagreement of the data in Table I. These are very large temperature gradients. If we assume a temperature of 5000° at a height of 800 km, we derive a temperature of 3100° (± 200°) at 1500 km. Or, assuming 4000° at 800 km, we derive a temperature of 2700° (± 150°) at 1500 km.

[28] *Monthly Notices of the Royal Astronomical Society*, **78**, 542, 1928.

Aside from being improbable, such temperature gradients are not borne out by the Ti II data. For it was found on page 10 that in the case of Ti II there is no appreciable dependence of log $E_h/E_{h+\Delta h}$ on excitation potential between 800 and 2000 km, and hence *no appreciable temperature gradient* at these heights. Menzel did not determine the temperature of the chromosphere from his Ti II data. We have examined both our material and his material for evidence of an excitation-potential effect in the flash intensities of the Ti II lines as compared with their Rowland intensities, and we find none. We conclude that if it exists at all it is certainly much less pronounced than in the case of Fe I. Hence the temperature gradient between the reversing layer and the chromosphere as determined by Ti II is certainly much smaller than that determined by Fe I.

Thus, we question the observational basis of Menzel's first and third conclusions in his summary on page 303, although we do not deny that according to current theories these conclusions seem probable.

Density gradients.—Menzel derives a composite law of density of emitting atoms from material which includes a number of different elements. Two determinations of the law as derived by two different methods are given in his equations (17.60) and (17.61). Below we rearrange them slightly in order that they may be more readily comparable with our formulae.

Material	Number of Emitting Atoms Proportional to:
Menzel composite:	
Formula 17.60	$e^{-2.79 \times x \times 10^{-8}} + 56e^{-6.06 \times x \times 10^{-8}}$
Formula 17.61	$e^{-2.47 \times x \times 10^{-8}} + 59e^{-6.41 \times x \times 10^{-8}}$ *
Mitchell and Williams:	
Ti II	$e^{-2 \times x \times 10^{-8}} + 60e^{-5 \times x \times 10^{-8}}$
Fe I of E.P. 1.0 volt	$e^{-3 \times x \times 10^{-8}} + 50e^{-6 \times x \times 10^{-8}}$

* The coefficient of the second term here does not agree with the coefficients in formula 17.61 as published. Professor Menzel has kindly advised us that the coefficients as published were in error.

The agreement is very satisfactory in view of the variety of elements which are involved in Menzel's composite law, and in view of the systematic differences between Menzel's heights and those of Mitchell.

Relative abundances.—It is difficult to criticize § 19 of Menzel's paper without appearing to cavil over small points. However, since the results of his paragraph depend on assumptions made with respect to small points, we shall have to consider them in detail.

We first consider the matter of departures from thermodynamical equilibrium. In the case of *Fe* I and many of the other well-represented elements, the lines of low excitation potential lie systematically to the violet of lines of higher excitation potential. This is well shown in our Table I. Hence, if the scale in the violet has been calibrated incorrectly relative to the rest of the spectrum so that lines in the violet appear systematically less intense than they should, then we would find relatively more atoms of high excitation potential than is actually the case. In a roughly calibrated spectrum it is very difficult, then, to separate errors in calibration from departures from thermodynamical equilibrium. Menzel's method of calibration is admittedly rough. He uses one calibration-curve (Fig. 18) for the region to the violet of λ 3750, and a second curve (Fig. 17) for the region to the red of λ 3885. We have attempted a rather more exact calibration but would hesitate to claim for it any great accuracy.

To find from Menzel's data the correction for values of excitation potential higher than 4 volts we shall consider the cases of *Mg* and of *Si* in detail. For some unexplained reason, Menzel did not use *Si* in determining this correction.

We take up *Mg* first, since, as Menzel says, more than half of the curve in Figure 47 (the right half) depends upon *Mg* alone. Now there is *effectively only one line of Mg* II *available* (a blend of two close lines at λ 4481). And in the case of *Mg* I, the value of log A_0 depends essentially on six lines of E.P. 2.70. Of these six, the three strong lines at $\lambda\lambda$ 3829–3838 practically determine log M, but all of these lines are too intense for his calibration-curve, the intensities of the lines being 20, 25, and 35, respectively, whereas the calibration-curve given in Figure 17 stops at intensity 19. Furthermore, we note that these three lines fall just at the end of the 1905 moving-plate spectrogram in a region where the focus is so poor that no "Intensities 1" are given in Table I. The foregoing intensities are taken from the 1905 stationary plate. But the calibration-curve in

Figure 17 supposedly applies to intensities taken from the 1905 moving plate. In fact, these lines lie just six-tenths of the way between the end of the region calibrated by Figure 18 and the beginning of the region calibrated by Figure 17; so that we should think a more appropriate value for log M than 4.6, which is given by Menzel, would be 5.4. This would correspondingly increase the value of log A_0 and hence the value of log M_c for the 3^1P^o-term of E.P. 4.33, which had been rather unsatisfactory before, since it was 0.2 less than the lower limit set by the observational data. Now from Menzel's equation (19.45) we have for Mg

$$\log A_1 = \log A_0 + \frac{5040}{T}\ (\text{Ionization level of chromosphere} - \text{I.P. of } Mg\ \text{I})$$

$$= \log A_0 + 1.064\ (9.5 - 7.61)\ ,$$

or

$$\log A_1 - \log A_0 = 2.0\ .$$

And by Menzel's equation (19.20) we have

$$\log A_0 = \log M_{2.70} + \frac{5040}{T}\ (2.70) - \log\ (3 \times 3) - \text{corr}_{2.70}$$

and

$$\log A_1 = \log M_{8.83} + \frac{5040}{T}(8.83) - \log\ (5 \times 2) - \text{corr}_{8.83}\ .$$

Substituting Menzel's value of 2.0 for log $M_{8.83}$ and the value 5.4 derived above for log $M_{2.70}$, we subtract these equations, and set their difference (log $A_1 - $ log A_0) equal to 2.0 as derived above. This gives

$$\text{corr}_{8.8} - \text{corr}_{2.7} = 1.1\ ,$$

which is to be compared with the value 1.9 given by Figure 47. Our argument is that, using Menzel's own calibration, we find from the Mg data that 1.1 is a rather better value than 1.9 which Menzel uses.

We now turn to the case of Si. On page 283 of Menzel's paper we read: "The low chromospheric abundance of Si, 60 times less than

that found by Russell for the reversing layer, *is real* and apparently inexplicable, though a small part of the discrepancy may possibly be attributed to Russell's assigning too much weight to the rather faint lines of Si II" (italics ours). In the chromospheric spectrum, however, these Si II lines are not faint. Moreover, since Menzel borrows Mitchell's intensities for other lines which are beyond his limits (for instance, the D lines of Na I at λ 5900 and the important Ca I lines at λ 6100), why does he neglect the Si II lines λ 6347 of intensity 8 and λ 6371 of intensity 7 in Mitchell's flash spectrum? These lines have an excitation potential of 8.1 volts. Their equivalent intensities on Menzel's scale, according to Menzel's Figure 16,[29] are 6 and 5, respectively. These intensities correspond to values of log T of 2.64 and 2.50, respectively (by his Fig. 17), so that the corresponding value of log M would be 2.9. Thus, by his Figure 47 and equation (19.20) we have

$$\log A_1 = 2.9 + 8.6 - 0.3 - 2.0 = 9.2 .$$

This value of log A_1 is 3.2 in excess of the value he computed from log A_0 for Si I, which latter, by the way, *was based on only one line.* The mean of these two determinations of log A_1 is 7.6. Now if log A_1 is 7.6 for Si, then the relative abundance of Si in the chromosphere is exactly as Russell found it in the reversing layer. Hence, we cannot agree that the deficiency to which Menzel refers is real.

Now we see no reason why the Si data should not also be used to determine the departure from thermodynamical equilibrium. By a procedure similar to that for Mg we find

$$\mathrm{corr}_{8.1} - \mathrm{corr}_{1.9} = 5.2 .$$

Thus, there is a discrepancy of about 4 volts between the corrections derived from the two sets of data, Mg and Si, which from a priori reasoning are of about equal weight! This not only affects the relative abundances of these two elements but also that of H and of Zn. If we apply a correction consistent with the one we have just derived from the Mg data, then the abundance of H in the chromosphere is

[29] In his Figs. 15 and 16 the headings for the ordinates and the abscissae have been accidentally reversed.

increased by a factor of 10 from the value Menzel gives in Table XIX. On the other hand, if we apply a correction consistent with the one we found from the Si data, the abundance is 10^4 times less than that in Table XIX. Thus there is a range of uncertainty of 10^5 in the abundance of H, and Menzel's remark that "hydrogen is fully 100 times more abundant than Russell's observations indicate" seems a bit unguarded.

It may be well to point out what seems to be an error in Menzel's abundance for La which has some bearing on the abundances of the rare earths in general. In his Table XVIII, the only observed value of log M for La is 3.2 for the a^3D-term of La II. The computed value set against this is 3.6. Of course the observed and computed values should agree when there is only one observed value to "fit." We have checked the observed value from his data and find it correct. This leads to a value of 2.3 instead of 2.7 for log A, and 3.7 instead of 4.1 for log T. Now Menzel's derivation of abundances for the rare earths rests on "a rather arbitrary analysis," the details of which are not given. The arbitrary analysis is necessary since the excitation and ionization potentials of most of the rare earths are still unknown. Inasmuch as the arbitrary analysis led to a value of 4.2 for the log T of La, which agreed within 0.1 with the value of log T he derived by the more rigorous method, he concludes that the approximate correctness of the arbitrary analysis is indicated. The error we have pointed out above spoils the very close agreement, although it is still good enough. He does not claim much accuracy for the values of log T for the rare earths, but, since on the average they exceed the relative log T found by Russell in the reversing layer by 1.3, he feels justified in regarding the rare earths as being considerably more abundant in the chromosphere than in the reversing layer. However, if we correct the values of log T for the rare earths in the chromosphere by the difference between the two determinations of log T for La, then they will exceed the values for the reversing layer by only 0.8 instead of 1.3. As to whether or not this difference of 0.8 may be regarded as a real systematic difference is an open question.

To sum up then, we would say that from Menzel's § 19 it is impossible to conclude more than that within the limits of error no

differences are detectable between the relative abundances in the chromosphere and those in the reversing layer, except that hydrogen is probably more abundant in the chromosphere. A comparison of Menzel's abundance estimates with those derived in this paper is given in the last column of Table IX.

We wish to thank Professor H. H. Plaskett and Dr. M. G. J. Minnaert for their very helpful suggestions and criticisms in connection with the first three sections of this paper.

FOUR STARS WHOSE SPECTRA HAVE
BRIGHT HELIUM LINES[1]

By PAUL W. MERRILL

ABSTRACT

Emission lines of *H*, *He* I, *He* II, *N* III, *C* III?, [*O* III], and the unidentified nebular lines λ 3869 and λ 3968 are present in the spectra of one or more of the faint stars AX Persei, RW Hydrae, CI Cygni, and MW 143. In the first three, bright λ 4686 *He* II is combined with dark bands of *TiO;* these three objects are variable in light and resemble Z Andromedae.

Radial velocities have been derived from the bright lines. The anomalous combination of bright lines requiring high excitation with dark bands of *TiO* appears to result from a definite set of conditions among which low pressure is probably essential.

This *Contribution* gives more detailed descriptions than have yet been published of the spectra of four stars with bright helium lines. The first three in Table I belong to that interesting group of objects whose spectra exhibit bright lines of ionized helium in combination with dark bands of titanium oxide, features which ordinarily occur only near opposite ends of the sequence of stellar temperatures.

TABLE I

FOUR STARS WHOSE SPECTRA HAVE BRIGHT HELIUM LINES

Star	R.A. 1900	Dec. 1900	Mag.	References	Remarks
AX Persei (MW 228)	1^{h}30^{m}0	+53°45′	9–13	*, †, ‡	*He* II and *TiO*
RW Hydrae........	13 28.8	−24 53	8–10	*, §	*He* II and *TiO*
CI Cygni (MW 232).	19 46.5	+35 26	10–12	*, †, ‖	*He* II and *TiO*
MW 143...........	4 11.6	+55 46	11.5	†	*He* I

* Paul W. Merrill and Milton L. Humason, *Publications of the Astronomical Society of the Pacific,* **44,** 56, 1932.

† Paul W. Merrill, Milton L. Humason, and Cora G. Burwell, Mt. Wilson Contr., No. 456; *Astrophysical Journal,* **76,** 156, 1932.

‡ E. M. Lindsay, *Harvard College Observatory Bu'letin,* No. 888, 1932.

§ Harlow Shapley, *ibid.,* No. 810, 1924.

‖ Annie J. Cannon, *ibid.,* No. 778, 1922.

AX PERSEI

An intense bright *Hα* line was discovered in the spectrum of this star on an objective-prism photograph taken on October 24, 1930,

by N. U. Mayall with the 10-inch refractor. Slit spectrograms have been obtained as tabulated.

Plate	Date	Dispersion at $H\gamma$
D VI 507	1931 Sept. 13	350 A/mm
C 5862*	Oct. 2	70
V 98	Nov. 9	120
C 6093	1932 Aug. 16	70

* Includes visual region.

As far as may be judged from plates with various dispersions and effective exposures, the spectrum was the same on all dates of observation. The following description depends largely on the last plate, which is the best of the series. The titanium oxide bands have about the intensity usually associated with class M3. The dark lines are not well shown because the continuous spectrum is narrow and underexposed, but they seem to indicate a type earlier than M3, say Kop, and to be less similar to dwarf than to giant spectra. The emission lines of hydrogen and of ionized helium (λ 4686) and several nebular lines are outstanding; those of *He* I and *N* III are less conspicuous, while numerous *Fe* II lines are barely visible. *C* III may be represented by a maximum near λ 4649. The estimated intensities of the bright lines are listed in Table II. In the visual region, plate C 5862 shows *H*α to be an extremely strong bright line; D3 of helium is conspicuous, and a trace of λ 6678 *He* is seen.

The radial velocity determined from the accurately measurable displacements of the emission lines is − 108 km/sec. The values for various elements as measured on three plates are in Table III. The numbers of lines measured are in parentheses. Plate C 5862 has small weight. The value derived by Humason from the low-dispersion plate D VI 507 is − 165 km/sec., which agrees within error of determination with the other values.

An investigation by E. M. Lindsay[2] based on the Harvard direct photographs of this star shows that it is a peculiar, irregular variable, having long periods of quiescence which are interrupted by well-marked oscillations with maxima at intervals of 600 or 650 days. The total observed range in photographic magnitude was from 9.4

[2] *Harvard College Observatory Bulletin*, No. 888, 1932.

PAUL W. MERRILL

to 13.4. The Mount Wilson spectroscopic observations were made during a nearly constant phase of magnitude 12.2.

TABLE II

INTENSITIES OF BRIGHT LINES

Element and Wave-Length	AX Per C 6093	RW Hyd C 5965	CI Cyg C 5879	MW 143 C 6155
H:				
3889.05............	1	tr	....	....
3970.08............	2	2	1	1
4101.74............	4	3	4	3
4340.47............	8	6	8	8
4861.33............	12	9	14	14
He I:				
3888.65............	....	....	....	2
3964.73............	....	....	....	tr
4026.19............	....	....	....	1
4120.81............	....	....	....	tr
4143.76............	tr	tr	....	tr
4387.93............	1	1	1.5	1
4437.55............	....	....	....	tr
4471.48............	1.5	0.5	1	4
4713.14............	0.5	tr	0.5	2
4921.93............	0.5	tr	0.5	bl
5015.68............	....	....	tr?	bl
He II:				
4199.87............	tr?	....	tr	....
4541.63............	1	tr	1	....
4685.81............	10	2	12	....
Nebular Lines:				
3868.74............	2	....	....	....
3967.51............	2	....	....	....
4363.21............	4	1	2	....
4958.91............	2	....	....	....
5006.84............	5	....	tr	....
N III:				
4097.33............	1.5	....	tr?	....
4634.14............	1	....	1	....
4640.63............	2	tr?	2	....
Fe II............	tr	....	tr	1

TABLE III

RADIAL VELOCITIES FROM BRIGHT LINES IN AX PERSEI
(km/sec.)

Plate	*H*	*He* I	*He* II	Neb.	*N* III	*Fe* II
C 5862......	−135(3)	−126(1)	(−166)(1)	−111(3)	....	....
V 98........	96(3)	110(2)	116 (1)	100(2)	....	....
C 6093......	−103(5)	−112(4)	−106 (2)	−115(5)	−99(3)	−102(6)

PLATE I

5007 — Hβ — 4686 — 4363 — Hγ — Hδ — He 3968 — 3869 —

a

b

c

d
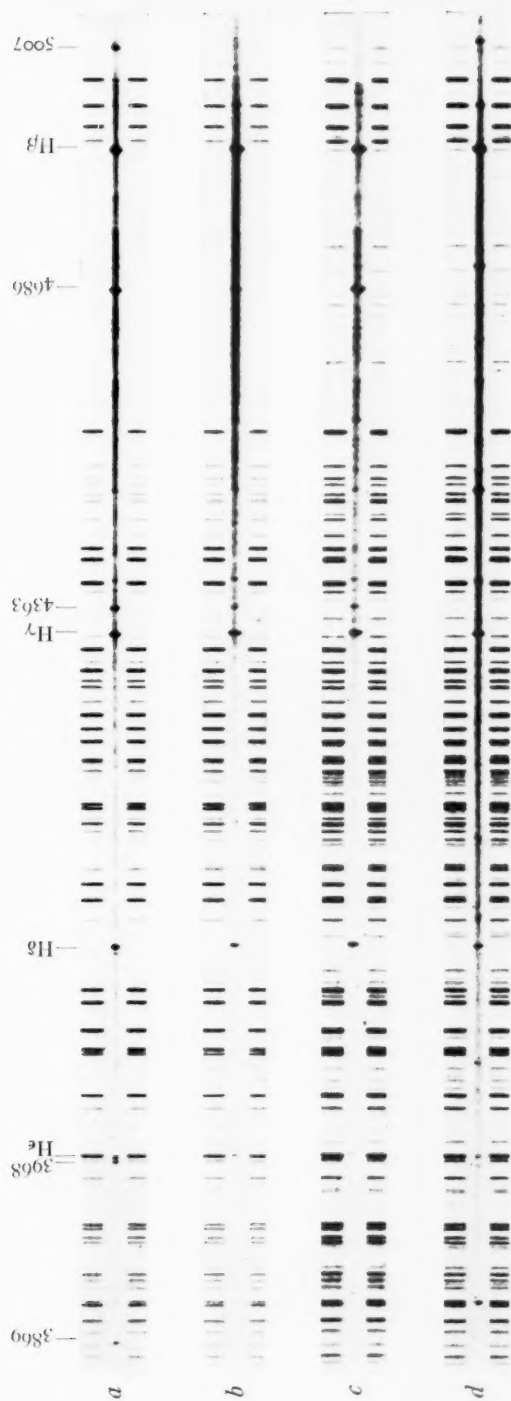
a) AX Persei, 1932 August 16
b) RW Hydrae, 1932 February 23
c) CI Cygni, 1931 October 28
d) MW. 143, 1932 October 11
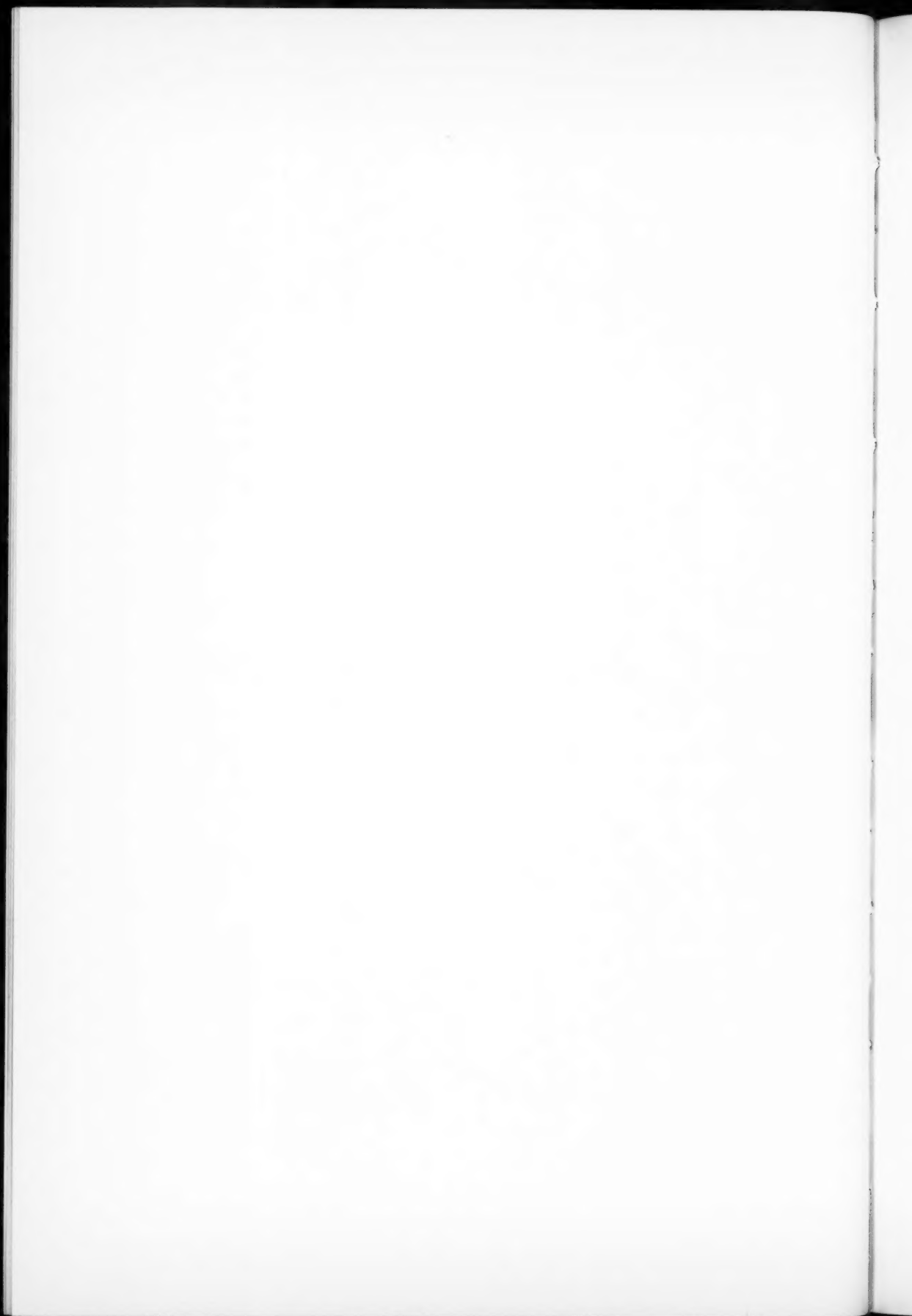
RW HYDRAE

Spectrograms of this object, which is included in the "Harvard Catalogue of Long-Period Variables,"[3] have been obtained in the program of radial velocities of long-period variables, on the dates shown. All have a dispersion at $H\gamma$ of 70 A per millimeter. The absorption-line spectrum is indistinct but appears to resemble that of AX Persei. The *TiO* bands have on the first and third plates an intensity corresponding to class M0; they are slightly stronger on the second plate, corresponding to class M1 or M2. Miss Cannon's classification of several Harvard objective-prism spectrograms ranges from K5 to M2.

Plate	Date	Approx. Vis. Mag.
C 5725	1931 May 8	9
C 5776	July 4	9.5
C 5965	1932 Feb. 23	8.5

The estimated intensities of the bright lines on the best plate, C 5965, are in Table II. The intensities of the hydrogen lines relative to the continuous spectrum are greatest on this plate and least on C 5776. Changes in the other bright lines, although much less pronounced, may be in the same direction.

The radial velocities derived from the bright lines on the three plates are $+23$, $(+8)$, and -7 km/sec., respectively. The second plate has low weight, but the difference between the first and the third may be significant.

A special study of the light-curve of this star was made by Professor Issei Yamamoto,[4] who found a mean period of 370 days. The mean photographic magnitudes at maximum and minimum were, respectively, 9.9 and 10.8, with extremes of 9.7 and 10.9. The light-curve is abnormal for a long-period variable.

CI CYGNI

Bright lines were detected in this object by Miss Cannon on an objective-prism plate made October 21, 1922, with the 16-inch Metcalf refractor of the Harvard College Observatory. The following data are from the *Harvard College Observatory Bulletin*, No. 778:

[3] *Harvard College Observatory Annals*, **79**, 161, 1928.

[4] *Harvard College Observatory Bulletin*, No. 810, 1924.

The spectrum closely resembles that of RS Ophiuchi, 174406, on July 15, 1898 (H.C. 76). The hydrogen lines $H\beta$, $H\gamma$, $H\delta$, $H\epsilon$, and $H\zeta$, are bright, and two other bright lines have the approximate wave-lengths 4686 and 4650. Several dark lines are also seen. An examination of more than three hundred photographs taken between 1890 and 1922 shows that the star is an irregular variable. The range is usually from about magnitude 11.5 to 12.2, but in July and August, 1911, the magnitude was 10.5.

Bright $H\alpha$ was found independently on an objective-prism plate taken at Mount Wilson on July 12, 1931. Slit spectrograms have been obtained on the dates shown.

Plate	Date	Dispersion at $H\gamma$
D VI 501	1931 Sept. 13	350 A/mm
C 5879	Oct. 28	70

The titanium oxide bands are decidedly stronger than in the spectra of RW Hydrae, and probably slightly stronger than in AX Persei, their intensity corresponding to class M4. The absorption lines are not well seen; they may resemble those of AX Persei, but are apparently less distinct.

In addition to very strong emission lines of hydrogen and ionized helium, numerous bright lines of lower intensity, due to He I, Fe II, N III, and [O III], are present. Traces of the forbidden iron lines [Fe II] and of the C III lines near λ 4649 are seen. The estimated intensities of the bright lines are in Table II.

The radial velocities derived from various groups of lines measured on plate C 5879 are as tabulated. The value derived by Humason from the low-dispersion plate D VI 501 is -40 km/sec.

Element	No. of Lines	Velocity
H	3	$+9$ km/sec.
He	7	0
Fe II	10	0
[O III]	2	-8
N III	2	$+2$
Weighted mean	. . . .	$+2$

MW 143

$H\alpha$ was seen as a strong bright line on objective-prism plates taken at Mount Wilson on September 27 and October 26, 1930. Slit spectrograms have been obtained on the dates as shown.

Plate	Date	Dispersion at $H\gamma$
D VI 527...............	1931 Oct. 14	350 A/mm
V 100................	Nov. 9	120
C 6155...............	1932 Oct. 11	70

This spectrum is quite different from the three just described. No bands are seen, nor absorption lines that are characteristic of any normal type. It is not certain that absorption lines are present, although a narrow minimum near λ 4427 looks like a real line. The continuous spectrum has a general intensity distribution resembling that of an A- or B-type star.

Another important difference is that, while several bright lines of neutral helium are conspicuous, λ 4686 of ionized helium is not seen. The estimated intensities of the bright lines are in Table II.

The radial velocities derived from the bright lines on plate C 6155 are as tabulated. The value derived by Humason from the low-dispersion plate D VI 527 is — 107 km/sec.[5]

Element	No. of Lines	Velocity
H.....................	4	—48 km/sec.
He....................	9	40
Fe II..................	17	—49
Weighted mean......	....	—46

REMARKS

The first three spectra bear a strong resemblance to one another and to Z Andromedae, described by H. H. Plaskett.[6] The spectrum

[5] The velocities from the low dispersion plates (D VI series) appear to require a systematic correction of approximately +50 km/sec. In proportion to the dispersion, this correction is of the same order as the corrections frequently applied to other series.

[6] *Publications of the Dominion Astrophysical Observatory*, **4**, 119, 1928. See also a note by Frank S. Hogg, *Publications of the Astronomical Society of the Pacific*, **44**, 328, 1932.

of MW 143, on the other hand, differs in the absence not only of
titanium bands, but of the bright lines of *He* II, *N* III, and [*O* III].
It begins to appear that the coexistence in the same spectrum of
these lines and of the titanium bands is not purely fortuitous, but
may arise systematically under certain stellar conditions. The pres-
ence of forbidden lines and the abnormally high intensity of *He*
λ 4387 relative to λ 4471 indicate low pressure,[7] which is probably
one of the essential conditions. The source of the high excitation[8] is
a most interesting question.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
October 1932

[7] Cf. the intensities of these lines in MW 143.

[8] The ionization potentials required to produce the ions *He* II, *N* III, and *O* III are
24.1, 29.5, and 34.9 volts, respectively, while the excitation required for the *He* II lines
is over 50 volts.

PLATE II
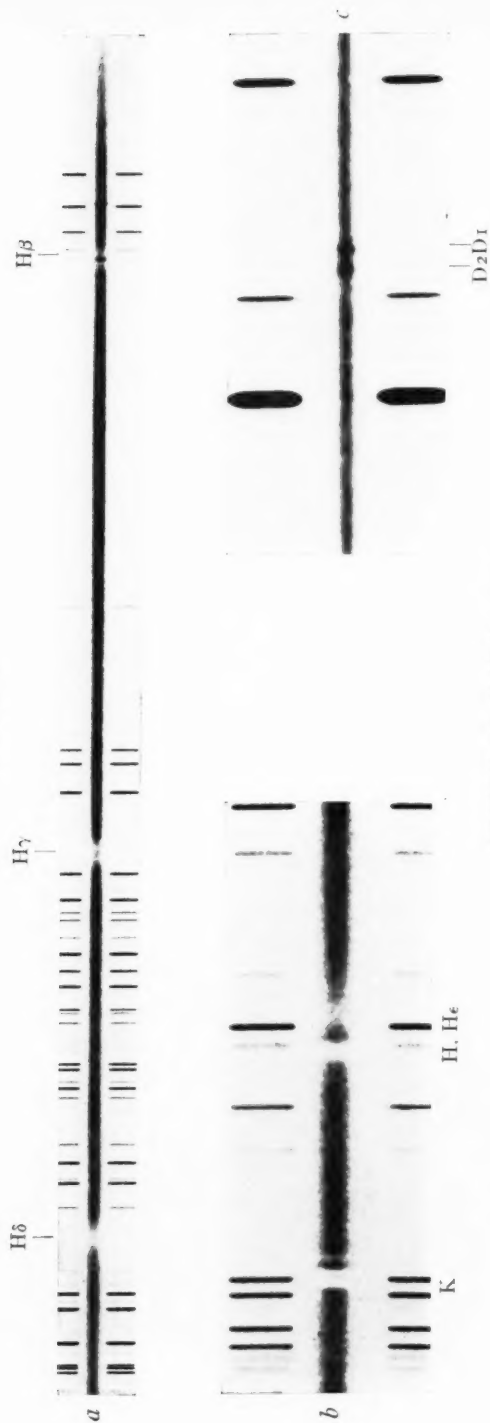
SPECTROGRAMS OF H.D. 190073

a) C5783. 1931 July 6
b) V205. 1932 July 10
c) G389. 1930 July 11

BRIGHT SODIUM LINES IN A
STELLAR SPECTRUM[1]

By PAUL W. MERRILL

ABSTRACT

This spectrum, type Aoep, has *bright sodium lines*, $D1,2$. Both H and K of calcium, in addition to a narrow dark line in the normal position, have an intense dark component 3 A wide, displaced 3.2 A toward shorter wave-lengths. The narrow lines have bright fringes on either side. Numerous enhanced metallic lines, most of them in emission, are visible. The radial velocity is about 0 km/sec. Neither the spectral details nor the integrated light have been observed to vary.

A bright $H\alpha$ line was detected in the spectrum of H.D. 190073[2] on an objective-prism photograph made July 6, 1927, with the 10-inch Cooke refractor on Mount Wilson. A spectrogram of the blue-violet region, taken with a slit spectrograph August 30, 1928, showed several bright lines of ionized iron in addition to central bright components at $H\beta$ and $H\gamma$. Helium lines were very weak or absent, while λ 4481 Mg II was a sharp absorption line, apparently without emission. Thus it was evident that the spectral type is approximately A0, decidedly later than that in which bright hydrogen lines are most frequent. Moreover, the spectrum appeared to differ somewhat from that of the normal emission-line star of its own type. These peculiarities were, however, relatively slight and of the same general nature as those frequently encountered when emission-line stars of early type are examined in detail. Later spectrograms, however, revealed features which set this star apart from all others whose spectra have been described. These are *bright sodium lines* and *H and K lines of calcium having abnormal intensity and structure.*

The sodium lines $D1,2$ are seen in emission on four spectrograms, C 5269, C 5311, G 389, and G 397, which include the yellow region. The lines are narrow and of moderate intensity (see Pl. II), but, on the grating plates, do not seem perfectly sharp. The intensities of the two lines appear nearly equal. Their displacements differ from those of the enhanced metallic lines, but agree approximately

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[2] B.D.+5°4393; R.A. 19ʰ58ᵐ1, Dec.+5°28′, 1900; mag. 7.9; spectrum A0.

with that of bright Ha (see Table II). Although traces of absorption are seen alongside the bright lines, the intensity is so low that interpretation is uncertain. H.D. 190073 is the only star, aside from

TABLE I

JOURNAL OF OBSERVATIONS

Plate	Date	Dispersion		Remarks
D 91*	1927 July 6	440 A/mm at	Ha	Objective prism
γ 16132	1928 Aug. 30	33	$H\gamma$	
C 5194	1929 May 23	38	$H\gamma$	
C 5221	June 21	38	$H\gamma$	
D 118*	July 11	440	Ha	Objective prism
C 5239	July 17	38	$H\gamma$	
C 5269	Aug. 17	38	$H\gamma$	Includes visual region
C 5306	Sept. 13	38	$H\gamma$	
C 5311	Sept. 14	38	$H\gamma$	Includes visual region
C 5330	Oct. 9	70	$H\gamma$	Ten-inch camera
C 5477	1930 June 12	38	$H\gamma$	Underexposed
C 5479	June 13	38	$H\gamma$	
G 389	July 11	66	. . .	Grating, visual region
C 5507	Aug. 6	38	$H\gamma$	
G 397	Aug. 8	66	. . .	Grating, visual region
C 5783	1931 July 6	38	$H\gamma$	
V 205	1932 July 19	23	K	Three-prism

* D 91 and D 118 are objective-prism photographs made with the 10-inch telescope. All other plates were with slit spectrographs attached to the 60-inch (γ, V) or the 100-inch (C, G) telescope; unless otherwise noted, they were made with one prism and an 18-inch camera and are of the blue-violet region.

TABLE II

DISPLACEMENTS OF LINES

Line	Character	Velocity	Total Number of Measures
		km/sec.	
Ha	Em.	+31	4
$D_{1,2}$ Na	Em.	+26	8
$H\beta$	Em.	+ 5	13
$H\gamma$	Em.	−25	13
Fe II	Em.	0	55
Ti II	Em.	+ 7	23
λ 4246 Sc II	Em.	− 2	4
λ 4481 Mg II	Abs.	− 6	12
λ 3933 Ca II	Abs.	0	10

novae, in whose spectrum bright sodium lines have been reported. In most emission-line stars of early type the sodium atoms in the strata above the photosphere are probably thoroughly ionized, too few remaining in the neutral state to produce observable D lines.

Calcium lines.—The ionized calcium lines H and K are very ab-
normal. K consists chiefly of two absorption components, one a nar-
row line of moderate intensity in its normal position, the other a
much stronger line 3 A wide, displaced 3.2 A toward shorter wave-
lengths. The central intensity of the undisplaced component is
about 70 per cent.[3] The edges of both components are sharply de-
fined, those of the displaced component giving an abrupt transition
from the continuous spectrum to an intensity so low that the plate
appears almost as clear as the general background (see Pl. II and
Fig. 1).
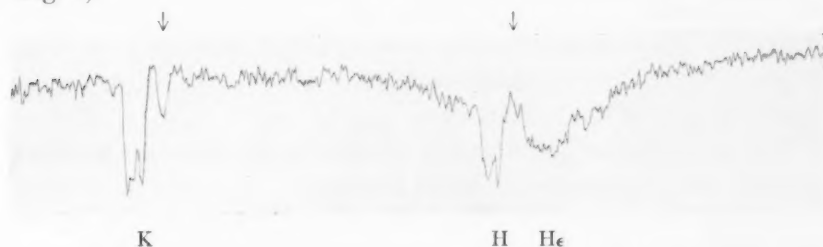
FIG. 1.—Microphotometer intensity-curve of the region of H and K from plate
V205 (see Plate II). The normal positions of these lines are indicated by arrows.

Between the two absorption components is a maximum whose
effective center lies 1.1 A toward shorter wave-lengths from the
normal position of K. Within the broad displaced component is a
very narrow weak maximum 2.9 A toward shorter wave-lengths
from the normal position of K. On the best plate (V 205) there is an
uncertain trace of another much weaker maximum 0.9 A farther in
the same direction.

H is confused by the superposition of *H*ε, but probably has
essentially the same structure as K. The strong displaced com-
ponent is well seen: the mean displacement of its center, measured
on nine plates, is 3.1 A, and, within errors of observation, its width
is the same as that of K. The other component is lost, on most
plates, in the *H*ε absorption, but on two plates a narrow minimum
was measured very nearly in the normal position of H.

Four facts make it probable that the narrow undisplaced H and

[3] Spectrograms of greater purity might yield a lower value.

K lines are of stellar origin: (1) the type, Ao, is one in which de-
tached lines are not usually found; (2) the velocity agrees with that
from stellar lines and (3) differs by 16 km/sec. from the solar motion
component, an unusually large difference for detached lines; (4) D1
and D2 are not seen as detached lines.

The presence of emission on either side of the dark lines in the
normal positions of H and K, suggested by the appearance of the
spectrograms, is confirmed by the photometric curve of plate V 205,
Figure 1. The maxima on either side of K rise above the level of
the continuous spectrum by perhaps 9 per cent. At H, the situation
is complicated by the superposition of $H\epsilon$, but any reasonable as-
sumption concerning the form of $H\epsilon$ calls for emission near H to
explain the observed intensity-curve. The width of the bright fringes
on either side of H and K is $1\frac{1}{2}$ or 2 A.

Hydrogen lines.—The emission components of the hydrogen lines
are centrally superposed on broad ill-defined absorption lines which
increase in intensity from $H\alpha$ to $H\zeta$. On an objective-prism plate,
D 91, the dark lines $H\zeta$ to $H\kappa$ appear approximately as strong as in
a normal Ao star. The central emission is very intense at $H\alpha$, con-
spicuous at $H\beta$, weak at $H\gamma$, and scarcely recognizable at $H\delta$. At
$H\gamma$ it appears indistinctly double.

Traces of absorption are seen on a few plates near the position
of helium λ 4471.

Enhanced metallic lines.—Narrow bright lines of ionized iron and
titanium are seen on practically all the spectrograms. Most of them
are of low intensity and offer little contrast to the continuous spec-
trum, although a few iron lines in the green are fairly conspicuous.
From 3 to 9 iron lines and from 0 to 5 titanium lines were measured
on each plate, not including λ 4549, which is probably a blend of
lines of both elements. The ionized scandium line at λ 4246 was
measured in emission on four plates; λ 4481 (ionized magnesium) is
a sharp absorption line without noticeable emission.

Displacements.—The enhanced metallic lines agree within errors
of determination in yielding a velocity of 0 km/sec., which probably
represents the star's radial motion. The lines include bright *Fe* II,
Ti II, *Sc* II, as well as dark *Mg* II and *Ca* II (narrow component).
The displacements of the bright hydrogen components exhibit a

strong progression from $H\gamma$ to $H\alpha$. The bright sodium lines agree approximately with $H\alpha$ (see Table II).

The spectrum does not appear to have varied during the interval covered by the Mount Wilson observations.

Magnitude.—The integrated light apparently is constant at magnitude 8.2 (photographic). The B.D. magnitude is 8.0. At my request Dr. Harlow Shapley kindly had the star examined on two hundred Harvard plates. He reports:

From these, 36 scattered throughout an interval of some 30 years were measured twice; they were the plates that on first examination seemed to show best evidence of variation. The magnitudes are as follows:

Mag.	Number
8.0	1
8.1	2
8.2	28
8.3	5

Mean.....8.2

In other words, there is no evidence of variation in excess of a tenth of a magnitude.

PHOTOMETRY OF THE GEGENSCHEIN

By C. T. ELVEY

ABSTRACT

The sky background in the region containing the *Gegenschein* has been measured with the photo-electric photometer attached to the 40-inch telescope of the Yerkes Observatory. These measurements furnish sufficient data to separate the illumination which is due to the *Gegenschein* from that which arises from the light scattered in the atmosphere. The observations were made on the nights of September 25, 28, and October 1, each at about six hours U.T., and the results are shown in the form of diagrams by plotting the observations and drawing the isophotes. The changing form and position of the *Gegenschein* is evident. The integrated brightness of the *Gegenschein* is equivalent to stars of photographic magnitudes +0.52, −0.06, and −0.28, respectively, for the three dates. The central brightnesses per square degree are, respectively, 6.54, 6.51, and 6.22 mag.

The Gegenschein was discovered three-quarters of a century ago by Theodor J. C. A. Brorsen,[1] but for thirty years very few observations were made and discussed, with the result that very little was known about it. Consequently, it was independently discovered by T. W. Backhouse[2] in 1868 in England and again in 1883 by E. E. Barnard[3] while observing at Nashville, Tennessee. Since then there have been many long series of observations of the position, size, and form of the Gegenschein, notably by T. W. Backhouse,[4] E. E. Barnard,[5] A. E. Douglass,[6] A. Searle,[7] and many others.

There have been very few photometric observations of the Gegenschein, owing, of course, to its low luminosity and large size. Photographs have been obtained by A. E. Douglass[8] with a specially constructed camera and multiple exposures and by Miss Margaret Harwood[9] with a short-focus lens attached to the 10-inch photo-

[1] *Astronomische Nachrichten*, **42**, 219, 1855.

[2] *Monthly Notices of the Royal Astronomical Society*, **36**, 46, 1875.

[3] *Sidereal Messenger*, **2**, 254, 1883.

[4] *Publications of the West Hendon House Observatory*, **2**, 97, 1902.

[5] *Popular Astronomy*, **7**, 169, 1899.

[6] *Ibid.*, **2**, 29, 1894; **5**, 178, 1897.

[7] *Annals of the Harvard College Observatory*, **33**, 15, 1900.

[8] *Popular Astronomy*, **23**, 601, 1915 (abstract).

[9] *Annual Report of the Mount Wilson Observatory*, p. 96, 1924.

graphic telescope at Mount Wilson. Apparently no photometric measurements have been made from these photographs. On March 20, 1931, C. Hoffmeister[10] obtained two photographs on the same hour circle but separated by about 10° in declination from which he was able to measure photometrically a meridional section of the Gegenschein. In a study of the southern Milky Way and the Zodiacal Light with a surface photometer C. Hoffmeister[11] gives measures and a diagram showing the isophotes for the Zodiacal Light and the Gegenschein. H. Thiele[12] in a paper at the meetings of the Astronomical Society of the Pacific reports on a photometric study of the Zodiacal Light and Gegenschein but does not give his methods. Presumably they are visual observations.

Last June I reported[13] on the detection of the Gegenschein with the photo-electric photometer of the Yerkes Observatory. I have resumed observing it since it has moved into a better position in the sky, making a much longer series of observations on each night in order to obtain the isophotes instead of a curve of intensities across one section of the Gegenschein. These observations were made in the same manner—that is, I obtained a measure of the sky brightness in regions free from stars at intervals of 5° along an hour circle. The area of the sky effective on the photo-electric cell is a circle 220 seconds of arc in diameter or 0.00294 square degree. In the regions outside of the Milky Way it is possible to find areas as large as this free from stars. The procedure is to record the rate of charging of the electrometer for the cell exposed to the sky, then to close the shutter and to obtain the combined rate of charging due to the dark and leakage currents. Also, the electrometer sensitivity is taken at each position. This gives data from which the rate of charging can be obtained for the illumination coming from a given area of the sky. Each observation thus obtained is the sum of the illuminations originating in our atmosphere, or scattered in the atmosphere from terrestrial lights and from the moon, and those general radiations coming from the region outside of our atmosphere. In order to

[10] *Astronomische Nachrichten*, **242**, 353, 1931.

[11] *Veröffentlichungen der Universitätssternwarte zu Berlin-Babelsberg*, **8**, Heft 2, 1930.

[12] *Publications of the Astronomical Society of the Pacific*, **33**, 217, 1921.

[13] *Astrophysical Journal*, **75**, 424, 1932.

eliminate the illumination originating in the atmosphere a plot is made of the observations obtained for a given hour circle, as shown in Figure 1. The ordinates are rates of charging of the electrometer in millivolts per second for the area observed and the abscissae are the declinations at which the observations were taken. As may be
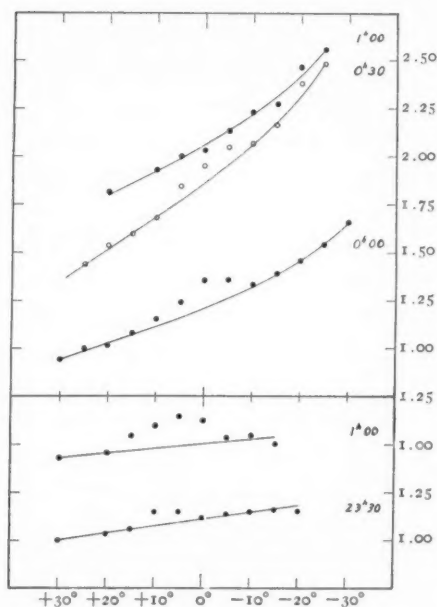

FIG. 1.—Plot of observations of the brightness of the sky along an hour circle. The deviations from the smooth curve are caused by the Gegenschein. The upper diagrams are for September 25 with the moon rising during the observations. The lower set is for October 1 when there was no moon in the sky.

seen from the figure, it is quite easy to sketch in the smooth curve which represents the change with declination of the illumination coming from the atmosphere. By taking the differences we then have the amount of the rate of charging of the electrometer which is due to the illumination coming from outside the atmosphere. These are then reduced to unit area, 1 square degree. It should be noted that in Figure 1 the upper diagrams are for September 25 when the moon was rising so that the sky illumination was becoming brighter. The observations were taken in order of right ascension. The lower diagrams are for October 1 and on that night there was no moon.

These values must still be corrected for the atmospheric extinction. I have employed the tables which we use for stellar observations, that is, the extinction, $E = 0^m.2\, f\, \sec z$, where f is a factor dependent upon the transparency[14] of the sky. Standard stars are observed also on each night for calibration. The two stars chosen are 4 β Trianguli, 2^h04^m, $+34°31'$, $m_p = 3.22$, A5; and 25 η Tauri,

[14] See a paper by Joel Stebbins, *Publications of the Washburn Observatory*, 15, 16, 141, 1930.

3^h42^m, $+23°48'$, $m_p = 2.85$, B5p. In each case the photographic magnitude has been used since it is very close to that determined photo-electrically. The error thus introduced is smaller than the uncertainty in the brightness of the Gegenschein. The standard stars have been introduced into the results only where the brightnesses have been given in magnitudes.

The results of three nights of observations are shown in Figure 2. On a chart of the right ascensions and declinations the values of the intensities in the Gegenschein have been plotted (as rates of charging of the electrometer per square degree), corrected for differential extinction. The isophotes have been sketched and the outermost one which represents zero intensity has been dotted since it is rather uncertain. The anti-sun has been plotted as a small black dot, and the ecliptic as a dotted line.

The Gegenschein on September 25 is more or less roundish with a tendency to be flat on the south side. Considering the Gegenschein as a whole the center is about 2° behind the anti-sun and a little south of the ecliptic. The intensity of the Gegenschein per square degree near its center is equivalent to a star of photographic magnitude 6.54 and the integrated intensity is equal to a star of magnitude $+0.52$.

The next observation was obtained three days later and the Gegenschein has changed in form in the interval, becoming elongated in the ecliptic. It still seems to center below the ecliptic, and if one considers only the isophote of greatest intensity, the center is ahead of the anti-sun; but if the entire Gegenschein is considered, the center seems to be at about the same longitude. The intensity per square degree near the center is equivalent to a star of photographic magnitude 6.51 and the integrated intensity is -0.06.

The last observation is of October 1, again an interval of three days. Here the form has a tendency to be "egg-shaped" with the longer axis along the ecliptic. The center is definitely ahead of the anti-sun in longitude, and perhaps a little north of the ecliptic. The intensity per square degree at the center is equal to a star of magnitude 6.22 and the total intensity is that of a star of magnitude -0.28.
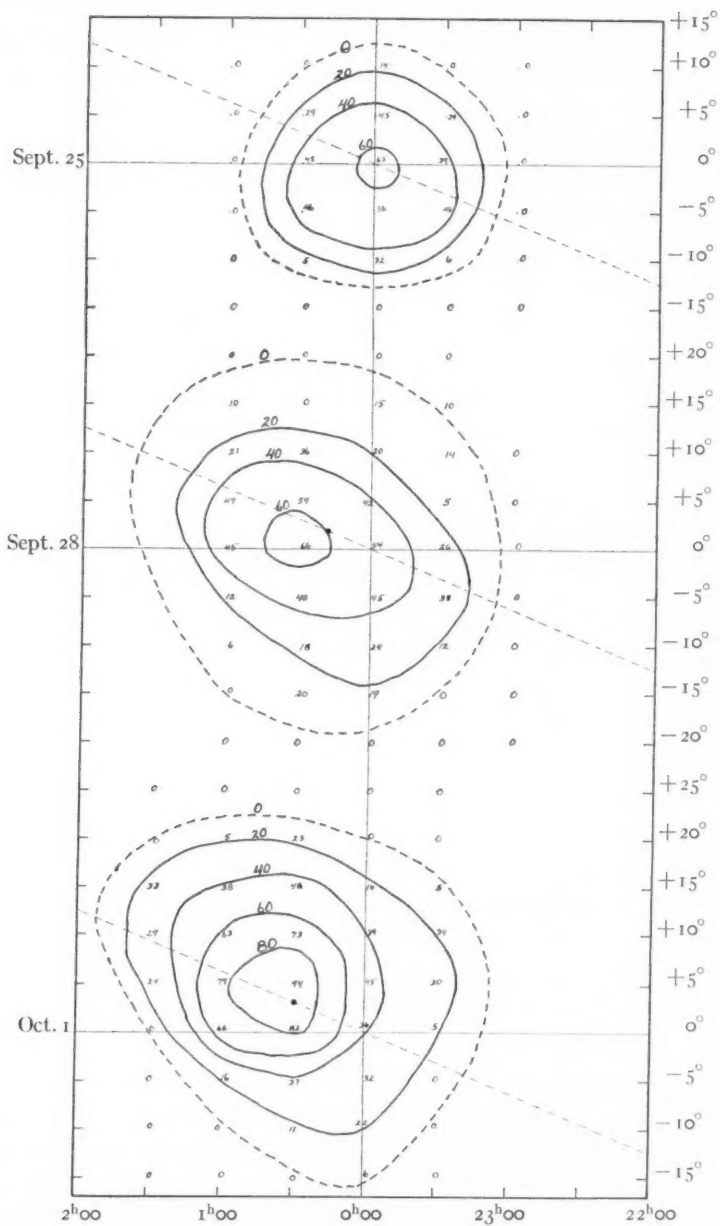
FIG. 2.—Isophotes of the Gegenschein made with the photo-electric photometer. The co-ordinates are right ascension and declination and the numbers in the diagram are rates of charging of the electrometer in millivolts per second per square degree. The dotted line is the ecliptic.

On the first two dates the central intensity is the same, but the size has increased from the first to the second, accounting for an increase in the total intensity of 70 per cent. On October 1 the area is about the same but the intensity has increased so that the total light has become 22 per cent greater than on September 28. The increase in total intensity from September 25 to October 1 is 0.8 mag., or by a factor of 2.1, or an addition of 110 per cent.

It is rather difficult to decide what limits to use in making a measure of the size of the Gegenschein, especially if these photometric observations are to be compared with visual ones. The isophote representing zero intensity is subject to rather large errors and the illumination may decrease much more gradually to zero. The isophote marked "20" (millivolts per second per square degree) would be a definite limit as long as the same instruments are used. Expressing the intensity corresponding to isophote of 20 mv/sec./ square degree in terms of stellar magnitudes, with the aid of the standard stars, so that other photometric measurements may be made comparable, we obtain for September 25, 7.82 mag.; for September 28, 7.83 mag.; and for October 1, 7.90 mag. The diameters of the Gegenschein are as shown in the accompanying tabulation.

Date	Diam. in Long.	Diam. in Lat.
1932 Sept. 25...............	22°	21°
Sept. 28...............	32	22
Oct. 1...............	35	26

It is generally believed that the eye can distinguish a difference of intensity of about 1 per cent above the background. If so, then one should be able to distinguish the Gegenschein out to the isophote corresponding to a rate of charging of the electrometer of 3.4 mv/sec./square degree. This would give diameters somewhat larger (by an average of 6°) than shown in the table. However, I am inclined to believe that with the intensity so near the threshold of visibility, the eye would not distinguish so small a variation. I did not make careful visual observations, but on the night of September

28 the Gegenschein did not appear to be more than half as large as is given in the tabulation.

I wish to express my appreciation to Mr. Paul Rudnick, Fellow in the Department of Astronomy, for his assistance in the reduction of the observations.

A NOTE ON THE VALUE OF THE GENERAL CONSTANTS OF THE SAHA EQUATION

By RAYMOND T. BIRGE

ABSTRACT

The two general constants of the Saha equation are evaluated from recent probable values of the various fundamental constants involved, and are found to be (5040.50 ± 0.69) °K·abs-volt^{-1}, and -6.48016 ± 0.00056, respectively.

The well-known Saha equation is usually written[1]

$$\log \frac{X}{1-X} = -\frac{5048I}{T} + 2.5 \log T - 6.5 - \log p_e ,$$

where X is the fraction of the atoms of any element ionized in a gas at absolute temperature T, I is the ionization potential in volts of the atoms, and p_e is the electron pressure in atmospheres. The number 2.5 comes from a consideration of degrees of freedom and is exact. The two general physical constants, to be evaluated in this note, are 5048 (to be denoted C_1) and -6.5 (to be denoted C_2).

The general constant C_1 does not appear explicitly in Saha's original article.[2] Saha wrote $\log K = -(U/4.571T)+$, etc. He stated that U equals 2.302×10^4 calories for one volt-electron, and he gave the origin of this last number. The value of the ratio $2.302 \times 10^4/4.571 \ (=5036=C_1)$ was written explicitly by Saha[3] first in 1922 and at the same time also by Russell.[4] The figure frequently used now seems to be 5048, although Russell[5] and presumably others have for the last few years been using 5040. I have, however, not noticed any direct statement of the relation of C_1 to the various fundamental constants involved, or of its present probable value and probable

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[1] See, for instance, Russell, Dugan, and Stewart, *Astronomy*, p. 569, 1927. For a general discussion of this equation, including more recent extensions and modifications, see A. Pannekoek, *Handbuch der Astrophysik*, **3**, 265–69 and 289, 1930.

[2] M. N. Saha, *Philosophical Magazine*, **40**, 472, 1920.

[3] *Ibid.*, **44**, 1128, 1922.

[4] *Astrophysical Journal*, **55**, 119, 1922.   [5] *Ibid.*, **70**, 51, 1929.

error. The same situation is true, in part, for C_2. Hence the following remarks may not be amiss.

The constant C_1 arises from taking the logarithm, to base 10, of the well-known Boltzmann factor $\epsilon^{-h\nu/kT}$. This gives $-M(h\nu/kT)$, where $M = \log_{10} \epsilon = 0.4342945$. We now wish to express energy in electron-volts, in place of $h\nu$ ergs. Hence our expression becomes $-MAI/kT = -C_1I/T$, where I is the energy in electron-volts, A is the number of ergs per electron-volt, and k is the Boltzmann constant. Substituting for A and k their values in terms of more fundamental constants, one gets

$$C_1 = \frac{AM}{k} = \frac{10^8 \cdot T_o \cdot F \cdot M}{\nu_n \cdot A_n}.$$

The symbols are those used by the writer[6] in 1929, and with the values of T_o, F, ν_n, and A_n adopted in that article (p. 59), one obtains

$$C_1 = \frac{(10^8)(273.18)(9648.9)(0.4342945)}{(22.4141 \times 10^3)(1.013249 \times 10^6)} = (5040.50 \pm 0.69) \ ^\circ\text{K} \cdot \text{abs-volt}^{-1}.$$

The stated unit, abs-volt^{-1}, corresponds to the colloquialism that I is the ionization potential in volts. The more precise statement is that I equals the energy of ionization in electron-volts, and the unit of C_1 is $^\circ$K$\cdot$abs-volt-electron^{-1}.

The numerical magnitude of the second general constant C_2 can be obtained most easily from the expression

$$C_2 = -2.5M + 1.5 \log m + \frac{S_o'M}{R_o'}$$

where $m =$ atomic weight of the electron, $S_o' =$ one form of the Sackur-Tetrode constant, and $R_o' =$ one form of the gas constant per mole. The values, and the probable errors, of m, S_o', and R_o' are given in GC, 1929 (pp. 62, 65, and 61, respectively). One cannot, however, calculate the probable error in C_2 by combining in the usual manner the probable errors in m, S_o', and R_o', since each of these three constants contains, in part, the same fundamental constants. To calcu-

[6] "Probable Values of the General Physical Constants," *Physical Review Supplement*, **1**, 1, 1929. (To be denoted hereafter as *GC*, 1929.)

late the probable error one must first express C_2 in terms of funda-
mental constants only. The result is

$$C_2 = \log \frac{1}{c^4}\left(\frac{8\pi^3 A_n^3 \nu_n^5}{(e/m)^3 F^5 T_0^5}\right)^{1/2}\left(\frac{e^4}{h^3}\right).$$

Each of these fundamental constants, with its probable error, is
given in GC, 1929 (p. 59). The various probable errors are now to be
compounded in the unsual way. However, in the case of e^4/h^3 the
proportional error is to be obtained by the rule given more recently
by the writer,[7] since the errors in e and h are not independent. The
final result is

$$C_2 = -6.48016 \pm 0.00056.$$

It is doubtless true that the experimental errors in contemporary
work involving C_1 and C_2 are so large that it is immaterial whether
one uses $C_1 = 5040.5$ or 5048, and $C_2 = -6.48016$ or -6.5. But there
are certain advantages in the use of a consistent set of constants,
fundamental and derived, such as those given in GC, 1929. As I
have noted more recently,[8] there seems to be no compelling reason
at the moment for a change in the 1929 values, and still more
recent experimental results have tended to confirm this opinion.

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   [7] *Physical Review*, **40**, 228, 1932 (see p. 261). The proportional error in e^4/h^3 adopted
here is that of solution k, i.e , 9.224×10^{-4}.

   [8] *Ibid.*, p. 260.

NOTES

THE WIDTH OF BRIGHT $H\alpha$ IN γ CASSIOPEIAE

ABSTRACT

The width of bright $H\alpha$ in γ Cassiopeiae is found to be about 8.7 A, confirming earlier measures by Merrill and by Curtiss, but disagreeing with recent results of Ambarzumian and Kosirev.

In order to investigate the question[1] whether the width of the emission line $H\alpha$ in γ Cassiopeiae is in harmony with the rotational hypothesis of bright lines in Be stars, I have measured three spectrograms recently taken by Mr. F. E. Roach with the new grating spectrograph constructed at the Yerkes Observatory and used in connection with the 69-inch reflector of the Perkins Observatory. The linear dispersion is 26.5 A/mm. The exposure time was short, in order not to overexpose the bright core of $H\alpha$. The average width is 8.7 A, which is in excellent agreement with the measures of Merrill[2] and of Curtiss.[3] It is therefore probable that the width of $H\alpha$ in γ Cassiopeiae does not vary by a large amount. Adopting the widths as given by Curtiss, $H\beta = 5.1$ A, $H\gamma = 4.4$ A, $H\delta = 3.9$ A, $H\epsilon = 3.5$ A, and $H\zeta = 3.7$ A, we find that the law $\Delta\lambda \propto \lambda$ is fulfilled for all Balmer lines. The straight line relating $\Delta\lambda$ to λ crosses the axis of λ near 2000 A instead of at zero. This discrepancy is probably caused by a systematic error in the measurements depending upon the intensity of the bright lines; the higher members of the series are faint and are therefore measured too narrow. There is no reason to doubt that the broadening of the bright lines is caused by Doppler effect.

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December 15, 1932

[1] O. Struve, *Astronomische Nachrichten*, **246**, 105, 1932; V. Ambarzumian and N. Kosirev, *Poulkovo Observatory Circular*, No. 1, 1932; *Astronomische Nachrichten*, **246**, 171, 1932.

[2] *Lick Observatory Bulletins*, **7**, 163, 1913.

[3] *Publications of the Observatory of the University of Michigan*, **2**, 24, 1916.

AN EMISSION LINE OF HYDROGEN
IN THE SPECTRUM OF RIGEL

ABSTRACT

The spectrum of β Orionis (B8p) shows a peculiar line at $H\alpha$. The absorption line is weak, and is shifted about 1.0 A toward the violet with respect to $H\beta$, which is normal. On the red side of absorption $H\alpha$ is a faint and narrow emission line which is shifted about 1.5 A toward the red from the normal position of $H\alpha$.

A spectrogram of Rigel (β Orionis, spectrum B8p) taken with the new grating spectrograph recently constructed at the Yerkes Observatory and used in connection with the 69-inch reflector of the Perkins Observatory shows a peculiarity in $H\alpha$. Its absorption line is much weaker than that of $H\beta$ and is shifted by about 1.0 A toward the violet with respect to the latter. On the red side of the absorption line at $H\alpha$ there is a faint, narrow emission line which is shifted by about 1.5 A toward the red from 6562.79, the normal wavelength of the hydrogen line.

The whole appearance of $H\alpha$ in Rigel agrees with that of the same line in the spectrum of α Cygni. The latter was first observed and measured by Miss E. Phoebe Waterman[1] at the Lick Observatory, and her results were later verified by C. D. Shane[2] and by W. H. Wright.[3]

According to Wright, the absorption line in α Cygni is shifted about 1.4 A toward the violet, and the emission line about 1.2 A toward the red from the normal position of $H\alpha$. Measures of the total absorption of $H\alpha$ in α Cygni by Elvey and Keenan[4] show that in this star, too, $H\alpha$ is much weaker than $H\beta$. In Rigel, as well as in α Cygni, $H\beta$ and the higher members of the Balmer series are strong, narrow lines devoid of the wide wings produced by ionic Stark effect.

There is a distinct similarity betweeen $H\alpha$ in these stars and the lines of P Cygni. But while in P Cygni the bright lines are practically undisplaced, they are displaced toward the red in Rigel and β Orionis. Nevertheless, it is probable that the origin of these lines is similar: The violet absorption line is presumably produced by an expanding atmosphere of hydrogen, and the bright line may originate

[1] *Lick Observatory Bulletin*, **8**, 1, 1913.

[2] *Ibid.*, **10**, 108, 1920.   [3] *Ibid.*, p. 109.

[4] *Astrophysical Journal*, **74**, 223, 1931.

in those parts of the gaseous envelope which are not projected upon the disk of the star. This phenomenon should give rise to an undisplaced emission line; but since in Rigel and in α Cygni this emission line must be superposed over a strong stellar absorption line produced in the reversing layer, it seems plausible that this superposition would produce an apparent shift of the bright line toward longer wave-lengths.

Rigel and α Cygni are both supergiants, of classes B8 and A2, respectively.[1] Bright lines similar to those of Rigel have been found, in the case of $H\alpha$, in ε Aurigae (F5p), by W. S. Adams and R. F. Sanford,[2] and, in the case of $H\beta$, in ι_1 Scorpii (cF5), in y Carinae (cF5), and in ε Aurigae, by P. Davidovich.[3] It should be noted, however, that we have failed to see bright $H\beta$ on any of the numerous spectrograms of ε Aurigae taken at the Yerkes Observatory. Bright $H\beta$ of the same general type was also observed in the spectrum of 17 Leporis.[4]

Since all the stars mentioned above are supergiants, it seems probable that lines of the P Cygni character are common in such stars. If our interpretation of these lines is correct, this would indicate that supergiants are frequently surrounded by very large and tenuous expanding envelopes.

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    December 29, 1932

[1] The designation "p" in the Harvard classification of Rigel refers to the sharpness and intensity of the absorption lines (*Annals of the Harvard College Observatory*, **28**, 184, 1901).

[2] *Publications of the Astronomical Society of the Pacific*, **42**, 203, 1930.

[3] *Harvard College Observatory Bulletin*, No. 846, 5, 1927.

[4] *Astrophysical Journal*, **76**, 85, 1932.

REVIEWS

Le Soleil. By GEORGES BRUHAT. Paris: Librairie Felix Alcan, 108 Boulevard Saint-Germain, 1931. Pp. 240; 15 plates and numerous diagrams. Fr. 20.

In this little paper-bound book Professor Bruhat of the University of Paris presents in non-mathematical form the subject matter of his course of lectures on the sun given at the Sorbonne in 1930. The subject is essentially modern knowledge of the physics of the sun, the chapter-headings comprehending solar radiation and temperature, direct observation of the photosphere and spots, the normal solar spectrum, the chromospheric spectrum, spectroheliographic studies, the corona, and electric and magnetic activity of the sun.

As might be expected, the effort to compress a large amount of material into such a small compass has led to ambiguity in places, but on the whole the author succeeds in presenting an authoritative picture that is surprisingly complete. In the section dealing with the chromosphere Milne's theories of radiative equilibrium are outlined, while the last chapter contains an excellent brief discussion of corpuscular radiation from the sun's surface.

Although written primarily to appeal to the non-technical reader who is interested in following the progress of astrophysics as applied to the sun, this handbook should be of chief value to the amateur observer with a reading knowledge of French. To the serious student a concise summary of this type will be of advantage, but the absence of precise bibliographical references is a serious limitation to its usefulness.

Bruhat deserves particular credit for confining his attention to direct observational evidence without wasting much time on the vague theoretical speculations that clutter the pages of many earlier books on the sun.

PHILIP C. KEENAN

Eclipses of the Sun. By S. A. MITCHELL. 3d ed. New York: Columbia University Press, 1932. Pp. 490. Numerous photographs. $5.00.

Comparison of this complete revision of Mitchell's well-known book with the first edition, published in 1923, serves to bring out strikingly the

accelerated activity of the past decade in attacks on the problems of the solar surface. Within this interval have appeared important observations of the flash spectrum by Davidson and Stratton and by Pannekoek and Minnaert, the numerous theoretical discussions by Milne and others on ionization and the equilibrium of the chromosphere, Russell's investigation of the constitution of the sun's atmosphere, the suggestions of Rosseland and of McCrea that the gases at the surface are in continual turbulent motion, the determination of the degree of this turbulence by Unsöld, the several independent measurements of the relative intensities of lines in the Fraunhofer spectrum in connection with the rapid accumulation of theoretical predictions of the distribution of energy within multiplets, Mitchell's studies of the correlation of heights with intensities in the flash spectrum, and, more recently, Menzel's elaborate discussion of the observations of eclipse expeditions from the Lick Observatory.

These and many other contributions have not only revolutionized current theories, but have introduced new problems which had not even been thought of in 1923. To summarize and discuss this mass of material has required a complete re-writing of those parts of the book which deal with the chromosophere and the corona. Mitchell has succeeded in carrying out this thorough revision without too greatly increasing the size of the volume. This has been accomplished in part by abridging some of the accounts of early eclipses, which have otherwise been left unchanged. On the whole it must be said that this compression effects an improvement in the style.

The chapter dealing with the verification of the theory of relativity by astronomical observations has been brought entirely up to date with a summary of the controversy of the past year between the Lick and Potsdam observers over the displacement of stars in the vicinity of the sun at the eclipses of 1922 and 1929. Mitchell accepts Trumpler's reductions and concludes that "the Einstein theory of relativity has been abundantly verified by astronomical observations." He thus considers the evidence now to be much stronger than that available in 1923, when he wrote that many more observations would be necessary to settle the status of the theory.

The new edition is not only very attractively printed and bound, but is also more compact than the earlier ones. Only in the reproduction of the photographs was the first edition superior, and there the difference is slight.

P. C. KEENAN

Handbuch der wissenschaftlichen und angewandten Photographie. By ALFRED HAY; continued by M. V. ROHR. Volume 6, Part I. Wien: Julius Springer, 1931. Pp. viii+289; Figs. 263, Pl. 2. Unbound, RM. 34; Bound, RM. 36.80.

The sixth volume of this treatise deals with the scientific applications of photography and includes in its first part the three subjects: "Stereophotography" by L. E. W. Van Albada (Amsterdam), "Astrophotography" by Ch. R. Davidson (Greenwich), and "Photographic Projection" by F. P. Liesegang (Düsseldorf).

The second part, in which we are more especially interested here, was translated from the English by W. E. Bernheimer (Vienna). It covers in 131 pages the great variety of applications that photography has found in the different branches of astronomy: in astrometry, in photometry, in spectroscopy, and in the study of individual objects—sun, moon, planets, and nebulae, comets being omitted curiously enough. The article is abundantly illustrated with diagrams and reproductions of astronomical photographs, but there are no bibliographic references except a few mentioned by the translator at the end of the article.

The sequence of the various chapters is somewhat unexpected: the author does not enter into the technique of photography but starts from the photographic refractor and the geometric problem of securing positions from plates, reference being made more particularly to the normal astrograph for the Carte du Ciel as used at Greenwich.

The next chapter deals with the Einstein-shift of stellar images in the sun's gravitational field. The Greenwich Observatory and especially the author have done creditable pioneering work in this line, but it seems odd that this very special and recent subject should be put in the foreground, preceding the chapters on stellar parallaxes, on photometry, and on spectroscopy, subjects which have been developing for many years into important new fields. In fact, they have grown so much that the limited space allotted for these subjects necessarily results in a somewhat too superficial treatment of the material. Other peculiarities in the arrangment of the material are noted when we find the chapters on the rotation of the planets and of the sun preceding those on the photography of the sun and then, most unexpectedly, a general chapter on the reflecting telescope before mentioning further photographic results on the moon, the nebulae, and the planets.

Aside from this unusual arrangement most of the material is presented in a very readable way. The practical acquaintance of the author with

many phases of the material presented results in close first-hand informa-
tion. His choice of photographic documents is excellent, and the reproduc-
tions are remarkably good on the paper used.

<div style="text-align: right">G. Van Biesbroeck</div>

The Stars of High Luminosity. By Cecilia H. Payne. New York:
  McGraw-Hill, 1930. Pp. xiii+320. Bound, $3.00.

This volume is number 3 in the series of "Harvard Monographs" of
which the preceding issues were *Stellar Atmospheres* (1925), by Miss
Payne, and *Star Clusters* (1930), by Professor Shapley. Miss Payne states
in the Preface that *The Stars of High Luminosity* is intended to replace
Stellar Atmospheres, since the latter volume is out of print and also some-
what out of date, due to the advances in astrophysics since 1925. She
also states a fact which is apparent after a comparison between the earlier
and later volumes has been made, i.e., the earlier picture of a stellar
atmosphere is simpler and more convincing than the later one. At the in-
ception of the ionization theory a large amount of evidence was found al-
most immediately which seemed to confirm the assumptions of the theory
completely, and it appeared at the time that the few outstanding anom-
alies would soon capitulate to the investigations which were being
carried on. At the date of publication of this book, however, and even
more so in the interval since 1930, serious difficulties have arisen, and at
the present time it is not possible to accept completely the explanations
of eight years ago.

The excellent agreement between the early theoretical work of Fowler
and Milne and the observed results made it appear likely that the theo-
retical and observational branches would proceed *pari passu,* but the
failure of more recent predictions to be checked observationally further
complicates the situation. Especially might be mentioned the attempts
made by Milne to predict absolute magnitude effects in stellar spectra.
His latest work on the subject has such complicated assumptions and
such widely varying results dependent on changes in the assumptions that
it is doubtful if any sort of a definite observational check can be made.

The question of the color-temperatures of the early B- and A-type
stars has also been a difficult problem. At the present time, however,
work that has been completed since the present volume appeared makes
it seem probable that this difficulty will be explained by the selective
absorption of light in interstellar space.

After a preliminary survey of the problems involved, the first part of

the book treats methods of spectrophotometry and the theoretical inter-
pretation of the forms of spectral lines. A general summary of data known
with regard to the c-stars is made. Then follow chapters in which normal
stars and supergiants of each spectral type are treated in detail. Next
come discussions of the cepheid variables, the intermediate variables hav-
ing periods between 50 and 100 days, and the long-period variables. The
final section of the book is concerned with a general analysis of stellar
atmospheres. An appendix gives a catalogue of c-stars.

The book covers so many different subjects that it hardly seems fair to
pick out an isolated statement or so for criticism, but mention might be
made of several points about which there seems to be some question.
In speaking of the determination of stellar temperatures Miss Payne
states that ionization theory has assigned the temperature of 10,000° to
class Ao. It should be mentioned that, *directly*, the Fowler-Milne ioniza-
tion theory is unable to assign any temperature to a star. It is only by
assuming likely values for the pressure in the star's atmosphere that we
can obtain a value for its temperature. The importance of determining
accurate color-temperatures for stars of classes Ao and later to be used as
standards of temperature cannot be overemphasized. So many anom-
alies have arisen in the attempt to classify the stars in a temperature
sequence merely by the intensities of certain lines in their spectra that
more direct methods are highly important.

There must be some error in the inclusion of 2 Monocerotis in the
catalogue of c-stars. It is also strange to see α Carinae included in the
same catalogue with the remark "Lines narrow—but no c-character."

There is an index and the volume is attractive as to appearance and
has been very carefully proofread.

                                                W. W. MORGAN

Strahlenoptik. By M. HERZBERGER. Berlin: Julius Springer, 1931.
   Pp. 196. Bound, RM. 19.40.

This is an unusually concise and elegant presentation of the subject
of geometric optics, developed *ab initio* from Fermat's principle of
shortest light path, using as tools the calculus of variations and vector
analysis, which are well adapted to the subject. The first order, or
Gaussian imagery, with its many important theorems, is well covered.
Forty-seven pages are devoted to the third-order imagery—a chapter of
utmost importance to the designer of lenses. A very useful historical chap-
ter and a copious bibliography conclude the book.

                                                F. E. ROSS

Physics of the Air. By W. J. HUMPHREYS. 2d ed. New York: Mc-
Graw-Hill Co., 1929. Pp. xii+654. $6.00.

The second edition of this book, although revised and enlarged, has
eleven pages fewer than the first edition. The enlargement has been ac-
complished through economy in the set-up of the volume, by increasing
the length of the line by 11 per cent and the length of the printed page by
more than 7 per cent, and by reducing several of the full-page illustrations
to about half a page. This economy has given a neater-appearing book,
except for the halftones, which were reduced in size. These are not of as
good quality as those of the first edition.

Many of the chapters have been revised and brought up to date,
especially those dealing with the circulation of the air. An additional sec-
tion on "Meteorological Acoustics" has been included in this edition.
This section is Part III and contains two chapters, one on "Meteorologi-
cal Effects on Sound" and the other on "Sounds of Meteorological Origin."
The first discusses the velocity of sound in air, the reflection and refrac-
tion of sound, and the location of sound (range-finding by sound); and
the other treats of such subjects as the creaking of the snow, howling of
the wind, humming of wires, thunder, etc.

C. T. ELVEY

Théorie de la quantification dans la nouvelle mécanique. By LOUIS
DE BROGLIE. Paris: Hermann et Cie, 1932. Pp. xxviii+250. Fr.
70 net.

Although the title might seem to imply a treatise on methods for the
solution of the wave equation, the present volume is primarily an exposi-
tion of the general theory of quantum dynamics. The purpose of the
author has been to develop this theory as far as possible from the stand-
point of wave mechanics, and there can be little doubt that this is the
method most suitable for a first approach to the subject.

Over a third of the book is introductory in character, and in it the wave
equation is first set up by means of the optical-mechanical analogy.
There follows a simple discussion of the dual nature of light and matter
as both corpuscular and ondulatory, in which it is clearly brought out that
the corpuscular aspect of optical interference requires a statistical rela-
tion between the wave-function and position which it is plausible to ex-
tend to the case of matter. The optical analogy also serves to introduce
a second principle, that of the spectral resolution of matter waves, which,
together with Planck's relation, gives the wave-mechanical method for

calculating the statistics of energy. The foundations are thus laid for the generalizations of the transformation theory which permits the prediction of the statistics of any dynamical quantity. This section contains also three chapters on the solution of the Schrödinger equation in special cases.

The second part of the text is devoted to the extended formulation of the theory, beginning with four chapters on necessary mathematical preliminaries—Hilbert space, function space, continuous matrices, and the Dirac δ-function. For the expression of the general principles the author has chosen the language of differential operators and their characteristic values and functions, rather than that of vectors and tensors in Hilbert space, thus connecting as closely as possible with the elementary notions of space-time waves for a single particle. Nevertheless, the methods of the matrix calculus are described in some detail, and the solution of a characteristic value problem for a differential operator is consistently regarded as analogous to a principal axis transformation. The discussion of continuous spectra is much more complete than in the majority of texts and forms a particularly valuable feature, although here, or elsewhere, many readers would probably welcome the inclusion of a greater number of specific applications of the theory.

The final portion of the book (about seventy pages) is given over to a somewhat abstract discussion of general statistical theorems, the uncertainty relations, the method of variation of constants, which is the only perturbation method considered in detail, adiabatic invariance, and the quantum-mechanical analogues to constants of motion in classical mechanics.

Paper, binding, and typography leave little to be desired.

F. C. Hoyt

Exposé électronique des lois de l'électricité. By Marcel Boll. Paris: Hermann et Cie, 1932. Pp. 72. Fr. 15 net.

All physicists and engineers would doubtless agree that an analysis of any electrical phenomenon will eventually lead to its atomic basis. This analysis is unavoidable, even for the technician, in dealing with such things as vacuum tubes and photo-electric cells, but it is by no means customary, even for the physicist, in the case of such familiar concepts as voltage, resistance, impedance, or the phase of an alternating current, except perhaps in a vague and indefinite way. As the author points out in his Preface, this is partly a matter of convenience and partly a matter of

tradition. It is nevertheless of importance to the physicist and of interest to the engineer to be able to understand long familiar, but none the less basic, phenomena in terms of atomic theory. The present volume should therefore appeal to all who are intimately concerned with electrical theory, for it is probably the first attempt at a consistent exposition of the engineering aspects of electricity from a purely electronic standpoint.

The treatment is simple but by no means non-technical and presupposes some familiarity with kinetic theory, and, of course, with conventional electrodynamics. The principal subjects treated are: metallic conduction and the heating effect of currents; circuits with induction and capacity; transformers; motors and generators; radio reception and transmission.

This highly original treatise in miniature should meet with a sympathetic reception from physicist and engineer alike. At the same time it is well not to lose sight of the fact that for detailed calculations—distribution of charge on a conductor, radiation from an antenna of given form—the Maxwell equations and "continuous fluid" theory are still essential.

F. C. Hoyt

L'idée générale de la mécanique ondulatoire et de ses premières applications. By MARCEL BOLL. Paris: Hermann et Cie, 1932. Pp. 74. Figs. 3. Fr. 15 net.

For the reader with an elementary knowledge of differential equations this little volume offers an attractive and stimulating introduction to wave mechanics. In the first three chapters the author describes briefly the formal aspects of the wave equation and its application to the hydrogen atom. The last two chapters are largely independent of the previous ones, and contain brief accounts of the theory of chemical valence and of electronic conduction in metals.

F. C. Hoyt